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CONSTRUCTION AND USE OF CHARTS IN DESIGN STUDIES OF GAS TURBINES

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SUMMARY

A method is presented for the computation and graphic representation of a series of possible turbine designs for any specific application. The use of a preliminary design chart is suggested for determining the number of stages and the effects of exit whirl and annular-area divergence on possible configurations. A specific design chart can then be made to aid in the study of the relations between turbine radius ratio, diameter, and significant design parameters such as Mach numbers, turning angles, and blade root stresses.

INTRODUCTION

In designing turbines for aircraft engines, it is desirable to obtain units having high reliability, light weight, low frontal area, simplicity, and high efficiency. Because the number of variables involved in designing turbines is large and the relations among them are complex, determination of the best design compromises may become quite involved. A method of computation and graphic presentation of a series of possible turbine designs for any specific application is presented herein to aid in determining these compromises.

Usually the first step in a design study is to obtain an estimate of the number of stages required, the type of velocity diagram to be used, and the need for annular flow-area divergence. A preliminary design chart is suggested for this purpose. Such a chart indicates whether a single-stage turbine will do the job with any specified centrifugal stress at the blade root, Mach number, and size limitations that may be imposed. The preliminary chart also indicates the effects of exit whirl and annular flow-area divergence. If a single-stage turbine is inadequate, each stage of a multistage unit can be checked against the design limitations so that an estimate can be made of the number of stages required.

With the information obtained from the preliminary chart, a specific design chart can be made for each stage of the required turbine. The specific design chart presents the parameters that describe the flow within the stage in terms of velocities and flow angles and relates these parameters to design variables such as size, radius ratio, centrifugal stress at the blade root, rotative speed, and specific weight flow. This chart contains sufficient information for fixing the dimensions of the unit.

The results obtained by the methods given in this report should be 2170 considered as preliminary in nature because of the simplifying assumptions that are used. These results serve to eliminate areas in which design limitations cannot be satisfied and indicate the points for which final analyses should yield suitable designs.

ANALYSIS

Assumptions. - The following assumptions are made herein: (1) one-dimensional flow occurs with no radial components, (2) all losses take place in the rotor, and (3) a constant value of the ratio of specific heats exists.

It is assumed that there are no radial or circumferential variations in one-dimensional flow. The value of the specific weight flow at one radius is considered to be representative of the values over the entire blade height. The effects of radial variations can be estimated from the charts for some types of design, specifically where constant specific work over the blade height is used. The assumption of no radial components requires that the flow entering a row of blades at any fixed radius leave that row at the same radius. The assumptions do not preclude the use of an annular-area convergence or divergence.

Most turbine stators have convergent flow passages with favorable pressure gradients; therefore, the flow coefficient is usually very close to 1.0 and the assumption that all losses occur in the rotor should not introduce appreciable errors. Boundary-layer accumulations can be taken into account by means of area changes to compensate for expected blockage. An efficiency value for the rotor process is assumed.

Radial shifts in weight flow, stator losses, and the appropriate variation of the value of the ratio of specific heats should be considered in the final design analysis.

Parameters. - The quantities required to fix the turbine design consist of (1) gas weight flow, (2) blade velocity or rotative speed, (3) inlet total temperature and pressure, and (4) required work output.

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The form in which these quantities are used should be dependent on the specific application. Where the values of the quantities listed are fixed, the charts indicate possible configurations that could do the required job. If, however, it is desired to conduct the design studies of the turbine with consideration of possible variations of these quantities, the charts can be adapted to relate turbine parameters to corresponding parameters describing the driven component. For example, where the turbine is to be coupled to a compressor, as in a turbojet engine, it is convenient to relate the parameters to the compressor, in that compressor designs usually tend to limit the range of possible turbine configurations. For such an application, the gas weight flow would be considered equal to the compressor air weight flow and a useful parameter would be gas weight flow corrected to compressor inlet conditions. The rotative speed of the turbine is related to that of the compressor; therefore, the compressor-tip speed could be used as a variable and the turbine speed may be expressed as a function of the ratio of the diameters of the two components.

Significant parameters that describe a design are the velocities and the angles comprising the velocity diagram. Specifically, for evaluation of designs, rotor-inlet and -outlet relative velocities and the stage exit velocity and flow angle are important. Velocity is usually given as a Mach number in studies of compressible-flow phenomena; in this report, however, velocity is expressed as a critical velocity ratio V/a_{cr} where a_{cr} is the critical sonic speed, a function of the stagnation temperature, and is defined as

$$a_{cr} = \left(\frac{2\gamma}{\gamma+1}\, gRT'\right)^{\frac{1}{2}} \tag{1}$$

(All symbols used in this report are defined in appendix A. The notation is illustrated in fig. 1.)

The critical velocity ratio will have a value of 1.0 when the velocity is such that the Mach number is equal to 1.0. For higher or lower velocities, the critical velocity ratio will differ somewhat from the Mach number. The relation between Mach number and critical velocity ratio is presented in reference 1 (equation (43)). A plot of Mach number against critical velocity ratio for a ratio of specific heats γ of 1.30 is presented in figure 2.

The specific weight-flow parameter used is defined as

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{w}{A\rho' a_{cr}} \tag{2}$$

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The annular-flow area A is defined as

$$A = \pi r_T^2 \left[1 - \left(\frac{r_h}{r_T}\right)^2\right] \tag{3}$$

Turbine-stress limits must be considered in the evaluation of designs. The stress parameter, a measure of the centrifugal stress at the blade root, can be expressed as

$$\frac{\sigma}{\tau \rho_b} = \frac{U_T^2}{2g144} \left[1 - \left(\frac{r_h}{r_T}\right)^2\right] \tag{4}$$

where σ is the centrifugal stress at the blade root, τ the taper factor, and ρ_b the density of the blade material. The derivation of this equation is presented in reference 2.

By combining equations (3) and (4) and using the relation $\omega r_T = U_T$, it can be shown that

$$\frac{\sigma}{\tau \rho_b} = \frac{\omega^2 A}{2\pi g144} \tag{5}$$

Thus, for constant rotative speed, the centrifugal stress is proportional to the annular-flow area. By substituting equations (1), (3), and (4) and the equation of state into equation (2), the following equation is obtained:

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{w}{\pi r_T^2} \frac{\rho_b}{\sigma/\tau} \frac{(U_T)^2 (T')^{\frac{1}{2}}}{p'} \left[\frac{R(\gamma+1)}{2\gamma g^3}\right]^{\frac{1}{2}} \frac{1}{288} \tag{6}$$

An adjusted specific weight-flow parameter F results if each of the quantities in equation (6) is divided by a representative value.

$$F = \frac{w}{\pi r_T^2} \frac{\rho_b/500}{\sigma/\tau/30{,}000} \left(\frac{U_T}{1000}\right)^2 \left(\frac{T'}{2500}\right)^{\frac{1}{2}} \frac{1}{p'/7000} \tag{7}$$

From equations (6) and (7)

$$F = K \frac{\rho V_x}{\rho' a_{cr}} \tag{8}$$

where the constant K is a function of γ, R, and the representative values in equation (7). For $\gamma = 1.30$ and $R = 53.3$,

$$F = \frac{64.29\ \rho V_x}{\rho' a_{cr}}$$

If the terms U_T, T', p', ρ_b, and σ/τ in the parameter F have the numerical values appearing in the denominator of their respective ratios, the entire parameter would be equal to the weight flow per unit turbine frontal area.

The terms of equation (7) that define the parameter F can be rearranged to suit specific applications. The form of equation (7) has been found useful in the study of turbines for turbojet engines. For other arrangements, the blade-speed and radius terms can be combined into an angular-velocity expression, the weight flow can be corrected to compressor-inlet conditions, the pressure can be expressed in terms of a compressor-pressure ratio, and so forth.

The adjusted weight-flow parameter F is used in the construction of the preliminary design chart herein, and the specific weight-flow parameter $\rho V_x/\rho' a_{cr}$ is used in the construction of the specific design chart.

The specific work parameter E is defined as

$$E = \frac{J\Delta h'}{a_{cr}^2} \tag{9}$$

where a_{cr} is based on the inlet total temperature. It can be demonstrated that across a single stage

$$E_{1,3} = \frac{1}{g} \frac{U}{a_{cr,1}} \left[\frac{V_{u,2}}{a_{cr,1}} - \left(\frac{V_{u,3}}{a_{cr,3}} \frac{a_{cr,3}}{a_{cr,1}}\right)\right] \tag{10}$$

Working equations. - In addition to the preceding equations, the following are pertinent working equations used in this report:

From the isentropic-flow relation and the general energy equation

$$\frac{\rho}{\rho'} = \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_{cr}}\right)^2\right]^{\frac{1}{\gamma-1}}$$

The specific weight-flow parameter (equation (2)) can also be written

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{V_x}{a_{cr}} \left\{ 1 - \frac{\gamma-1}{\gamma+1} \left[\left(\frac{V_x}{a_{cr}}\right)^2 + \left(\frac{V_u}{a_{cr}}\right)^2 \right] \right\}^{\frac{1}{\gamma-1}} \tag{11}$$

It is obvious that

$$\left(\frac{V}{a_{cr}}\right)^2 = \left(\frac{V_x}{a_{cr}}\right)^2 + \left(\frac{V_u}{a_{cr}}\right)^2 \tag{12}$$

and that

$$\alpha = \tan^{-1} \frac{V_x}{V_u} \tag{13}$$

A plot of the relation of equation (11) to (13) is presented in figure 3. This plot, referred to as a "flow chart", presents the specific weight-flow parameter plotted against the critical velocity ratio V/a_{cr} with lines of constant axial and tangential components and flow angle α. A large working plot of figure 3 for $\gamma = 1.30$ is inserted in the back of this report.

The assumption of isentropic flow in the stator results in the following relations:

$$\left.\begin{aligned} T_1' &= T_2' \quad \text{thus} \quad a_{cr,1} = a_{cr,2} \\ p_1' &= p_2' \quad \text{and} \quad \rho_1' = \rho_2' \end{aligned}\right\} \tag{14}$$

The continuity equation through a turbine stage may be written

$$\rho_1 V_{x,1} A_1 = \rho_2 V_{x,2} A_2 = \rho_3 V_{x,3} A_3$$

When the parameter F defined in equations (7) and (8) is used, the following equation is obtained:

$$\frac{F_2}{F_1} \frac{A_2}{A_1} = \frac{\rho_1' a_{cr,1}}{\rho_2' a_{cr,2}}$$

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From equation (14),

$$F_1A_1 = F_2A_2 \tag{15}$$

and for constant area,

$$F_1 = F_2$$

Application of the parameter F to the continuity condition across the rotor yields the following result:

$$\frac{F_3A_3}{F_2A_2} = \frac{\rho_2'a_{cr,2}}{\rho_3'a_{cr,3}} \tag{16}$$

Generalization of equation (16) yields

$$\frac{F_nA_n}{F_1A_1} = \frac{\rho_1'a_{cr,1}}{\rho_n'a_{cr,n}} \tag{17}$$

Equation (17) may also be written

$$\frac{\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_n}{\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_1} \frac{A_n}{A_1} = \frac{\rho_1'a_{cr,1}}{\rho_n'a_{cr,n}} \tag{17a}$$

Adiabatic efficiency is defined as

$$\eta = \frac{\frac{T_1' - T_3'}{T_1'}}{1 - \left(\frac{p_3'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}} \tag{18}$$

By rearranging terms in equation (18) and introducing the value of E as defined in equation (9) and the relations described by equation (14), it can be shown that

$$\frac{p_1'}{p_3'} = \frac{p_2'}{p_3'} = \left[\frac{\eta}{\eta - 2\left(\frac{\gamma-1}{\gamma+1}\right)gE_{1,3}}\right]^{\frac{\gamma}{\gamma-1}} \tag{19}$$

Similarly,

$$\frac{a_{cr,3}}{a_{cr,2}} = \frac{a_{cr,3}}{a_{cr,1}} = \left(\frac{T_3'}{T_1'}\right)^{\frac{1}{2}} = \left[1 - 2\left(\frac{\gamma-1}{\gamma+1}\right)gE_{1,3}\right]^{\frac{1}{2}} \tag{20}$$

Generalization of equation (20) yields

$$\frac{a_{cr,n}}{a_{cr,1}} = \left[1 - 2\left(\frac{\gamma-1}{\gamma+1}\right)gE_{1,n}\right]^{\frac{1}{2}} \tag{21}$$

A plot of this function for values of $\gamma = 1.40$ and 1.30 appears in figure 4.

From equations (16), (19), (20) and the equation of state

$$\frac{F_3}{F_2}\frac{A_3}{A_2} = \frac{\rho_2' a_{cr,2}}{\rho_3' a_{cr,3}} = \frac{\rho_1' a_{cr,1}}{\rho_3' a_{cr,3}} = \left[\frac{\eta}{\eta - 2\left(\frac{\gamma-1}{\gamma+1}\right)gE_{1,3}}\right]^{\frac{\gamma}{\gamma-1}} \left[1-2\left(\frac{\gamma-1}{\gamma+1}\right)gE_{1,3}\right]^{\frac{1}{2}} \tag{22}$$

A plot of this relation for values of the adiabatic efficiency η of 0.90 and 0.85 and for γ of 1.30 and 1.40 appears in figure 5.

For any value of the tangential component, there exists a maximum value of the specific weight-flow parameter (see fig. 3). This relation can be found by taking the partial derivative of equation (11) with respect to the axial component V_x/a_{cr} and setting it equal to zero. The axial component of the critical velocity ratio for this condition is

$$\frac{V_x}{a_{cr}} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_u}{a_{cr}}\right)^2\right]^{\frac{1}{2}} \tag{23}$$

By using the relation between a and a_{cr} (reference 1, equation (29)) it can be shown that this condition corresponds to

$$V_x = a \tag{24}$$

At this point, the passage is said to be choked.

The following equation is obtained by substituting equation (23) into equation (11):

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_{max} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_u}{a_{cr}}\right)^2\right]^{\frac{1}{2}} \left\{1 - \frac{\gamma-1}{\gamma+1}\left[1 + \frac{2}{\gamma+1}\left(\frac{V_u}{a_{cr}}\right)^2\right]\right\}^{\frac{1}{\gamma-1}} \tag{25}$$

A plot of this relation for $\gamma = 1.30$ appears in figure 6.

Required velocities are found by solving the right-triangle relations involved in the velocity diagram (fig. 1). In computing the rotor inlet relative velocity, and its expression as a critical velocity ratio, the value of the critical sonic speed should be based on the relative stagnation temperature.

The static temperature at any station may be written in terms of relative stagnation temperature and relative velocity or in terms of absolute stagnation temperature and velocity:

$$T = T'' - \frac{W^2}{\frac{2\gamma}{\gamma-1}\,gR} = T' - \frac{V^2}{\frac{2\gamma}{\gamma-1}\,gR}$$

By use of this relation and equation (1), the ratio of relative critical sonic speed to the absolute value at any station is found to be

$$\frac{a''_{cr}}{a_{cr}} = \left\{1 - \frac{\gamma-1}{\gamma+1}\left[\left(\frac{V}{a_{cr}}\right)^2 - \left(\frac{W}{a_{cr}}\right)^2\right]\right\}^{\frac{1}{2}} \tag{26}$$

From the velocity diagram (fig. 1(b))

$$\frac{W}{a_{cr}} = \left[\left(\frac{V_x}{a_{cr}}\right)^2 + \left(\frac{V_u}{a_{cr}} - \frac{U}{a_{cr}}\right)^2\right]^{\frac{1}{2}} \tag{27}$$

Substitution of equation (27) into equation (26) yields

$$\frac{a''_{cr}}{a_{cr}} = \left\{1 - \frac{\gamma-1}{\gamma+1}\left[\frac{2\,V_u U}{a_{cr}^{\ 2}} - \left(\frac{U}{a_{cr}}\right)^2\right]\right\}^{\frac{1}{2}} \tag{28}$$

PRELIMINARY DESIGN CHART

The purpose of the preliminary design chart is to aid in the determination of the number of stages that will be required and to demonstrate the influence of exit whirl and annular-area divergence on possible designs.

Description

A typical preliminary design chart is shown in figure 7(a). The chart consists of two parts; the left side is a plot of the parameter F against the tangential component of the critical velocity ratio V_u/a_{cr} with lines of constant axial component of the critical velocity ratio V_x/a_{cr}. This section of the chart is plotted by use of equations (8) and (11). Lines of constant flow angle α are determined by use of equation (13). An extension of the chart for negative values of the tangential component can be added. For that region, the flow angle would be greater than 90°. Such an extension would be perfectly symmetrical with the region of positive tangential component; it is therefore omitted. If negative values are desired, the supplements of the flow angles that appear on the plot must be used. The left portion of the preliminary design charts thus applies to any station between blade rows in a turbine.

A line of maximum value of the ordinate can be drawn as an envelope to the curves. This line, shown as the dashed curve on figure 7(a), can be computed from equations (8) and (25). All physically possible points will lie below the envelope. The portion of the supersonic domain with axial components of the critical velocity ratio greater than 1.0 are omitted for clarity.

The right side of the preliminary design chart is a plot based on equation (22), which relates the values of the parameter F before and after the rotor in terms of the specific work parameter E for a constant area configuration and an assigned value of the adiabatic efficiency.

The charts presented in figure 7 were constructed for a ratio of specific heats of 1.30 and an adiabatic efficiency of 0.90.

Applications

The starting point in any analysis depends upon the factors that are assigned or limited.

Single-stage turbine with zero exit whirl. - For a design having zero exit whirl, the flow at the exit of the rotor corresponds to a point on the left ordinate where $\left(\frac{V_u}{a_{cr}}\right)_3 = 0$. For a typical case in which sufficient information is supplied to define the parameter F at the inlet to the turbine, it can first be assumed that the annular-flow area across the stator is constant, and $F_1 = F_2$. For the assigned blade speed, inlet total temperature, and required specific work, equation (10) can be solved for the tangential component at station 2, $V_{u,2}/a_{cr,1}$. The values of F_2 and $V_{u,2}/a_{cr,1}$ define a point on the left side of the preliminary design chart (fig. 7). By using the right side and the assigned value of E, the value of F_3 can be found for any assigned area ratio A_3/A_2. This application is illustrated in the following example:

Illustrative example I - Zero-exit-whirl design. - For this problem, the following assumptions were made:

(a) Weight flow per unit turbine frontal area, 20 pounds per second per square foot

(b) Required specific work parameter, 0.0140

(c) Maximum tolerable stress, 40,000 pounds per square inch

(d) Taper factor, 1.0

(e) Blade density, 500 pounds per cubic foot

(f) Turbine-inlet total temperature, 2500° R

(g) Turbine-inlet total pressure, 5120 pounds per square foot absolute

(h) Tip speed, 1000 feet per second

Assignment of the parameters in this fashion is representative of a turbine design for jet-engine application, where the air weight flow per unit frontal area and tip-speed values are based on compressor limitations. If it is assumed that the compressor and the turbine have the same tip diameter or any assigned ratio of diameters, the conditions specified for the compressor inlet can be readily reduced to corresponding values at the turbine inlet.

If it is assumed that $A_1 = A_2$ under the given conditions, F_2 can be computed from equation (7). For this example, it will be found that

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$$F_2 = 20 \frac{500/500}{40{,}000/30{,}000} \left(\frac{1000}{1000}\right)^2 \left(\frac{2500}{2500}\right)^{\frac{1}{2}} \frac{7000}{5120} = 20.5 \text{ (lb/(sec)(sq ft))}$$

The following is obtained by using equation (1) and the given inlet total temperature and blade speed:

$$\frac{U_T}{a_{cr,1}} = \frac{1000}{2200} = 0.454$$

When equation (10) is solved for $\frac{V_{u,2}}{a_{cr,1}}$ with $\frac{V_{u,3}}{a_{cr,3}} = 0$,

$$\frac{V_{u,2}}{a_{cr,1}} = \frac{32.2 \times 0.0140}{0.454} = 0.993$$

With these values of F_2 and $V_{u,2}/a_{cr,1}$, the point on the preliminary design chart corresponding to station 2 can be located. This example is worked out on the preliminary design chart shown in figure 7(b); the lines of constant angles and the envelope have been omitted for clarity. The value of F_3 for constant A can be found by drawing a horizontal line from the value of F_2 into the right section of the plot to the value of $E = 0.0140$. F_3 can be read on the abscissa or graphically transferred back to the left side of the figure by means of the reflecting line that appears on the right side of the preliminary design chart. The point corresponding to rotor-exit conditions is designated as station 3 on figure 7(b). The exit critical velocity ratio is about 0.68. If that value is thought to be high, it can be reduced by a divergence of the annular-flow area to give the desired exit-velocity ratio. For example, if a value of 0.60 is desired, the exit flow will be represented by point 3(a) on figure 7(b). The area divergence can be readily computed from the ratio of the values of F as

$$\frac{A_{3,a}}{A_3} = \frac{F_3}{F_{3,a}} = \frac{35.2}{32.9} = 1.07$$

The increase in area for constant rotative speed results in a stress increase as shown by equation (5). The new value is 40,000X1.07 or 42,800 pounds per square inch.

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The rotor-inlet critical velocity ratio can be obtained from its components by means of equation (12)

$$\frac{V_2}{a_{cr,2}} = \left[\left(\frac{V_x}{a_{cr}}\right)_2^2 + \left(\frac{V_u}{a_{cr}}\right)_2^2\right]^{\frac{1}{2}} = \left[(0.610)^2 + (0.993)^2\right]^{\frac{1}{2}} = 1.16$$

By means of equation (27), the rotor-inlet relative critical velocity ratio can be found from the known axial and tangential components and the blade-speed ratio as

$$\frac{W_2}{a_{cr,2}} = \left[(0.610)^2 + (0.993 - 0.454)^2\right]^{\frac{1}{2}} = 0.814$$

This value can be corrected for the value of the relative critical sonic speed by use of equation (26).

$$\frac{W_2}{a''_{cr,2}} = \frac{W_2}{a_{cr,2}} \frac{a_{cr,2}}{a''_{cr,2}} = \frac{0.814}{\left\{1 - 0.130\left[(1.16)^2 - (0.814)^2\right]\right\}^{\frac{1}{2}}} = 0.852$$

Single-stage turbine with exit whirl. - As a starting point it is desirable to specify maximum tolerable exit conditions for configurations with exit whirl. These conditions can be specified in terms of exit axial component $V_{x,3}/a_{cr,3}$ and the flow angle α_3. As was previously mentioned, for negative tangential components, the flow angle will be the supplement of the value shown on the chart. The centrifugal stress at the blade root based on the exit conditions can be computed. The point corresponding to station 3 can be located and then the conditions at station 2 can be found by drawing a horizontal line to the reflecting line, a vertical line down to the line corresponding to the desired specific work parameter E, and then a horizontal line to the value of the inlet tangential component $V_{u,2}/a_{cr,1}$, which can be found from equation (10). The ratio $a_{cr,3}/a_{cr,1}$ of equation (10) can be computed from equation (20) or read from figure 4.

The effect of area divergence through the rotor can be studied by adjusting the ratio F_3/F_2 in accordance with equation (16).

Illustrative example II - Exit-whirl design. - If the value of the rotor inlet relative velocity found for example I is considered to be

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too high, a reduction can be obtained by use of exit whirl. A design is studied in this example for which the rotor-exit axial component is limited to a value of 0.60 with an exit angle α_3 of 120° in which case the tangential component will be negative. The supplement of the angle is 60° and the point corresponding to these exit conditions has been marked as station 3 on figure 7(c). The value of F_3 can be read as 30.8. Under the turbine-inlet conditions given in example I, F_2 is equal to 20.5; and for $E = 0.0140$, the exit F based on constant area was equal to 35.2. In order to get the value of F at the exit down to 30.8, an area divergence of 35.2/30.8 or 1.14 will be required with a resulting stress of 45,600 pounds per square inch. The exit tangential component $V_{u,3}/a_{cr,3}$ is 0.35 (fig. 7(c)) and the required tangential component at station 2 can be found by use of a form of equation (10):

$$\frac{V_{u,2}}{a_{cr,2}} = \frac{32.2 \times 0.0140}{0.454} - 0.35 \times 0.940 = 0.664$$

The values of F_2 and $V_{u,2}/a_{cr,2}$ locate the point on figure 7(c) marked station 2. For this point, the rotor inlet relative critical velocity ratio $W_2/a''_{cr,2}$ is found to be 0.485, a considerable reduction over example I.

Multistage applications. - Preliminary design charts can be used in determining the number of stages to be used in the design of multistage turbines. The last stage of such a machine usually has the highest stress; it is therefore suggested that the possible design range of that stage be considered first.

The preliminary design charts handle one stage at a time, and the parameters involved are based on conditions at the inlet to that stage. The over-all specific work must equal the sum of the stage outputs. Because the specific work parameter is based on the inlet temperature of each stage, the following equation must hold:

$$E_{1,n} = E_{1,3} + E_{3,5}\left(\frac{a_{cr,3}}{a_{cr,1}}\right)^2 + E_{5,7}\left(\frac{a_{cr,5}}{a_{cr,1}}\right)^2 + \ldots + E_{n-2,n}\left(\frac{a_{cr,n-2}}{a_{cr,1}}\right)^2 \tag{29}$$

The last term of equation (29) can also be written

$$E_{n-2,n}\left(\frac{a_{cr,n-2}}{a_{cr,1}}\right)^2 = E_{n-2,n}\left(\frac{a_{cr,n}}{a_{cr,1}}\right)^2\left(\frac{a_{cr,n-2}}{a_{cr,n}}\right)^2$$

The specific weight-flow parameters between stages are related as shown by equation (17):

$$\frac{F_nA_n}{F_1A_1} = \frac{\rho_1{}'a_{cr,1}}{\rho'_n a_{cr,n}} \tag{17}$$

The right side of equation (17) is a function of $E_{1,n}$ (see equation (22)). The exit conditions can thus be computed for any assumed area ratio and turbine efficiency for known turbine-inlet conditions. With limiting values of the exit velocity set, the required area changes can be computed. The point that corresponds to such exit conditions can be located on the preliminary design charts, and conditions at the inlet to the stage can be obtained by working backwards on the chart. For such an analysis, the specific-work output across the stage must be considered variable. For an assumed value of $E_{n-2,n}$, the stage may be studied by use of equation (10). The value of $U/a_{cr,n-2}$ may be computed from the known inlet conditions by setting

$$\frac{U}{a_{cr,n-2}} = \frac{U}{a_{cr,1}}\left(\frac{a_{cr,1}}{a_{cr,n}}\right)\left(\frac{a_{cr,n}}{a_{cr,n-2}}\right) \tag{30}$$

The ratio $a_{cr,1}/a_{cr,n}$ may be evaluated from the known value of $E_{1,n}$ and $a_{cr,n}/a_{cr,n-2}$ from the assigned value of $E_{n-2,n}$.

When a design has been found such that the size, stress, and critical velocity ratios are satisfactory, the value of E for the stage is fixed. By subtracting that value from the over-all specific work and making an adjustment for the inlet-total-temperature changes, the remaining specific work can be computed from

$$E_{1,n-2} = E_{1,n} - E_{n-2,n}\left(\frac{a_{cr,n-2}}{a_{cr,n}}\right)^2\left(\frac{a_{cr,n}}{a_{cr,1}}\right)^2 \tag{31}$$

The process outlined can be repeated for other stages.

<u>Illustrative example III - Multistage-turbine design.</u> - For this example, the following assumptions were made:

(a) Specific work parameter, 0.0314

(b) Maximum tolerable stress, 30,000 pounds per square inch

(c) Taper factor, 1.0

(d) Blade density, 500 pounds per cubic foot

(e) Turbine-inlet total temperature, 2500° R

(f) Turbine-inlet pressure, 23,000 pounds per square foot absolute

(g) Maximum tip speed, 1000 feet per second

The specific weight flow is considered variable. If it is desired to have the last stage operate with zero exit whirl and a maximum critical velocity ratio of 0.6, the point representing the exit flow can be located on the preliminary design chart. This example is illustrated in figure 7(d), where the exit point is marked as station n. The value of F_n is equal to 32.8. For a constant-area design, the value of F at the inlet to the turbine can be found from the right side of the preliminary design chart for the assigned value of E or computed from equation (22); for this problem, the computed value of F is 8.52. For this value of F and other assigned factors, the weight flow per turbine frontal unit area can be found from equation (7) to be 28.0 pounds per second per square foot.

In considering the last stage only, and starting with the exit point, a trial value of $E_{n-2,n} = 0.015$ is assumed. With no increase in tip diameter, the blade speed based on conditions at the inlet to the stage can be found from equation (30) and by use of either equation (21) or figure 4.

$$\frac{U}{a_{cr,n-2}} = \frac{1000}{2200}\left(\frac{0.935}{0.858}\right) = 0.495$$

The required tangential component at station n-1 can be found from a form of equation (10).

$$\frac{V_{u,n-1}}{a_{cr,n-1}} = \frac{32.2 \times 0.015}{0.495} = 0.976$$

This point is designated as station n-1 in figure 7(d). The value of F_{n-1} is equal to 18.2. Velocity diagrams can be computed for this design. The remaining specific work can be found from equation (31)

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$$E_{1,n-2} = 0.0314 - 0.015 \left(\frac{0.858}{0.935}\right)^2 = 0.0188$$

A one-stage design with exit whirl will be sought for this value. The tolerable exit tangential component will be set such that the flow angle α_3 equals 120°. For this value, the tangential component is negative and α equals 60° (the supplement of the desired value) will be used on the preliminary design chart. Also, the exit axial component of the critical velocity ratio will be limited to 0.60. The design of this stage is illustrated in figure 7(e), where the exit point is station 3. The values of F_3 and $V_{u,3}/a_{cr,3}$ are 30.9 and 0.35, respectively. By working backwards with the value of $E_{1,3}$ equal to 0.0188, the value of F_2 is found to be 14.5. If the tip speed of the first stage is assumed equal to the tip speed of the second and equation (10) is solved with the use of figure 4, the required value of the inlet tangential component is obtained:

$$\frac{V_{u,2}}{a_{cr,1}} = \frac{32.2 \times 0.0188}{0.454} - 0.35 \times 0.917 = 1.01$$

This point is marked as station 2 in figure 7(e). The velocity diagrams can be computed for that design. If this one-stage design is not satisfactory, a two-stage configuration might be tried so that an over-all three-stage turbine would result. Alternatively, a redesign of the last stage might be considered in light of the study of the first stage. When satisfactory velocity diagrams have been obtained, the ratio of the values of the parameter F_3 actually used to the values required by equation (17) will indicate the required annular-area divergence. In the illustrative example just worked, the value of F_3 required by equation (17) is 18.2, but the flow at station 3 is set such that the value of F_3 actually used is 30.9. In order to obtain this condition, the area of the first stage must be reduced by the ratio $18.2/30.9 = 0.589$ of the area of the second stage.

In this example, a constant tip diameter and therefore a constant tip speed was assumed. The designer is free, however, to specify how the necessary area divergence is to be accomplished.

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SPECIFIC DESIGN CHARTS

When the general type of turbine has been fixed in a manner such as has been described in the preceding section, a specific design chart can be used to aid in fixing the turbine dimensions. The specific design charts must be made up individually for each required stage-specific-work output. The area ratio across the stage and the efficiency must also be known or assumed. Each chart also represents a fixed type of velocity diagram. The three types of chart discussed in this report are:

Type A, Zero-exit whirl

Type B, Fixed rotor-exit critical velocity ratio with variable exit whirl

Type C, Fixed rotor-inlet relative critical velocity ratio with variable exit whirl

Description

Representative charts for these three types appear in figure 8. The equations necessary for the construction of the charts are presented in appendixes B to E. The main body of the charts is a plot of the specific weight-flow parameter ahead of the rotor against the blade-speed parameter. Lines of constant value of critical velocity ratios such as rotor-inlet relative velocity, rotor-exit relative velocity, and axial velocity components are contained on the plot. The impulse line, which separates the region of positive reaction from the region of negative reaction, has been included in the plots, as well as a line for α_2 equals 20°. Definite envelopes exist for portions of the curves. The equations defining these envelopes are discussed in the appendixes. The envelopes indicate the regions on the charts in which all the possible turbine designs for the assigned conditions lie. However, practical considerations may impose limits which lie below the envelopes.

In the zero-exit-whirl chart (fig. 8(a)), the ordinate is a function of the exit critical velocity ratio for any given area ratio; in the chart for exit whirl with fixed exit velocity (fig. 8(b)), the ordinate is a function of the exit flow angle for an assigned area ratio. In both cases an auxiliary scale can be plotted along the ordinate.

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Because conditions at the blade root are generally the most critical, the abscissa for the representative figures is taken as the blade speed at the root of the blade. For given values of the inlet total temperature, the abscissa can be related to radius ratio, tip speed, and stress, and such a side plot can be added. If the inlet total pressure is also known, the ordinate can be related to the weight flow. In the representative charts, the weight flow has been utilized in the form of a weight flow corrected to compressor-inlet conditions divided by the turbine frontal area based on tip diameter:

$$\frac{W\sqrt{\theta_c}/\delta_c}{\pi r_T^2}$$

The equations for computing the side plots are given in appendix E. For any application, the form in which such side plots can be most advantageously utilized is dependent on the form in which design conditions and limits are set.

Applications

The specific design charts lend themselves to several possible applications depending on the nature of the specific problem. For example, desirable points can be marked on the charts, and values of speed and weight flow can be computed from the abscissa and the ordinate. By use of the side plots, designs for limiting stresses and desired weight flows or other combinations of the parameters may be investigated.

Estimation of radial variations in flow. - An estimate of the velocity diagrams at other radii can be made by use of the charts for designs having constant specific work output over the blade height. For designs which also specify constant specific weight flow, the charts will directly give the velocity diagrams at other radii as radial variations will occur along a line of a constant value of the ordinate. The abscissas of the points that correspond to flow at other radii can be obtained from the values of the blade-speed ratio at those radii.

For other designs with constant specific-work output over the blade height, only an estimate can be made. For example, for free vortex, simple radial-equilibrium designs, the axial components are constant over the blade height. Lines of constant axial component at station 2 have been added in figure 8(a). The velocity diagrams at other radii can be obtained by using the abscissas of the points that correspond to flow at other radii and by moving along a line of constant axial component at station 2. It will be noted that as such radial variations are studied on figure 8(a), the value of the exit axial critical velocity ratio will not remain constant as is required for free-vortex, simple radial-equilibrium design because of the assumption that no radial shifts of mass flow take place in the stage. In addition, an integration of the specific weight flow over the blade height will be in error because of the one-dimensional flow assumption.

Illustrative example IV - Applications of specific design charts. - Assume that study is required of possible turbine designs for a jet-powered airplane to fly at the following conditions:

Compressor inlet total pressure, lb/sq ft abs.	898
Compressor inlet total temperature, $^{\circ}R$	471
Compressor pressure ratio.	6
Compressor enthalpy rise, Btu/lb	87.0
Turbine inlet total temperature, $^{\circ}R$	2500
Turbine inlet total pressure, lb/sq ft abs.	5120

For these conditions, the value of the specific work parameter E for the turbine is 0.0140. This value is used in constructing the representative charts (fig. 8). Constant annular area and an efficiency of 0.90 were assumed. The side plot below the abscissa is computed for the given turbine inlet total temperature and an assumed blade density of 490 pounds per cubic foot.

A series of possible designs will be sought that will enable driving a compressor having a tip speed of 1065 feet per second and a corrected weight flow per unit turbine frontal area of 30 pounds per second per square foot. This example is illustrated in figure 9, in which the zero-exit-whirl configuration was used. The basic chart is identical to that presented in figure 8(a), except that the axial-component lines have been omitted for clarity. A line corresponding to a tip speed of 1065 feet per second has been added to the side plot below the abscissa. The locus of points common to this speed line and the line in the ordinate side plot, which has a value of 30 pounds per second per square foot, are then plotted for equal values of the radius ratio. This line is marked 1065,30 in figure 9; it represents possible turbine designs having the same tip diameter as the compressor. Possible turbine designs that have tip diameters larger than the compressor can be similarly plotted because the corresponding turbine tip speed will vary directly as the ratio of turbine- to compressor-tip diameters, and the weight-flow term varies inversely as the square of the ratio. Curves of turbines having tip speeds of 1200 and 1400 feet per second and thus weight flows of 23.6 and 17.4 pounds per second per square foot, respectively, have been plotted on figure 9. In addition, the lines of constant radius ratio have been added.

As the 1065 line lies outside of the envelope of the curves, no designs can be obtained in that region because the areas will not pass the required air weight flow. Possible designs can be studied by considering the desirable rotor inlet and exit critical velocity ratios, stress, degree of reaction, and other factors.

If a desirable point of operation is selected that does not fall on one of the plotted tip-speed curves, the expansion, or contraction of the turbine diameter required can readily be found. A line of known tip speed and weight flow $\left(\frac{U_T}{a_{cr}}\right)^*$ and $\left(\frac{W\sqrt{\theta_c}/\delta_c}{\pi r_T^2}\right)^*$ is plotted in figure 10. A horizontal line corresponds to constant annulus area, a vertical line to constant radius, and the point m to a known point on the curve; the required values of tip speed and weight flow at point n can be obtained by using the following equations:

$$\frac{U_{T,n}}{U_{T,m}} = \frac{r_{T,n}}{r_{T,m}} = \left\{1 - \left(\frac{r_h}{r_T}\right)_m^2 \left[1 - \left(\frac{U_n}{U_m}\right)^2\right]\right\}^{\frac{1}{2}} \tag{32}$$

and

$$\left(\frac{w\sqrt{\theta_c}/\delta_c}{\pi r_T^2}\right)_n = \frac{\left(\frac{w\sqrt{\theta_c}/\delta_c}{\pi r_T^2}\right)_m}{\left(r_{T,n}/r_{T,m}\right)^2} \tag{33}$$

The radius ratio at point n can be determined from

$$\left(\frac{r_h}{r_T}\right)_n = \left\{1 - \left(\frac{r_{T,m}}{r_{T,n}}\right)^2 \left[1 - \left(\frac{r_h}{r_T}\right)_m^2\right]\right\}^{\frac{1}{2}} \tag{34}$$

CONCLUDING REMARKS

A method has been presented for the computation and graphic presentation of a series of possible turbine designs for any specific application. A preliminary design chart is used to obtain an estimate of the number of stages required, the type of velocity diagram, and the need for annular flow-area divergence. This chart is general and can be used for a preliminary study of any design.

The specific design chart is constructed with the information obtained from the preliminary design chart. The area ratio, the specific work, and the efficiency must be known or assumed before this chart can be constructed. The specific design chart relates the flow within the stage in terms of critical velocity ratios and flow angles to design variables, such as size, radius ratio, centrifugal stress, and rotative speed. The principal use of the design charts is to present the relations among the design variables in such a way as to facilitate the determination of the best design compromises necessary to obtain the desired configuration.

Because of the assumptions made, the information obtained from the charts should be considered as preliminary in nature, and a more accurate final design analysis should be made.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, March 22, 1951.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A annular flow area, sq ft

a velocity of sound, ft/sec

E work parameter, $\frac{J\Delta h}{{a_{cr}}^2}$, sec^2/ft

F adjusted specific weight-flow parameter,

$$\left(\frac{w}{\pi r_T^2}\right)\left(\frac{\rho_b/500}{\sigma/\tau/30,000}\right)\left(\frac{U}{1000}\right)^2\left(\frac{T'}{2500}\right)^{\frac{1}{2}}\frac{1}{p'/7000}\text{, lb/(sec)(sq ft)}$$

g gravitational constant, 32.2 ft/sec^2

h specific enthalpy, Btu/lb

J mechanical equivalent of heat, 778 ft-lb/Btu

K constant

p absolute pressure, lb/sq ft

R gas constant, ft-lb/(lb)($^{\circ}$R)

r radius, ft

T absolute temperature, $^{\circ}$R

U blade velocity, ft/sec

V absolute velocity of gas, ft/sec

W relative velocity of gas, ft/sec

w weight-flow rate of gas, lb/sec

α flow angle of absolute velocity measured from tangential direction (fig. 2), deg

β flow angle of relative velocity measured from tangential direction (fig. 2), deg

γ ratio of specific heats

Δ prefix to indicate change

δ pressure reduction ratio, p/p_0

η adiabatic efficiency

θ temperature reduction ratio, T/T_0

ρ gas density, lb/cu ft

ρ_b density of blade material, lb/cu ft

σ centrifugal stress at blade root, lb/sq in.

τ taper factor, ratio of σ in tapered blade to σ in untapered blade

ω angular velocity, radians/sec

Subscripts:

c compressor-inlet stagnation conditions

cr critical, state at speed of sound

h station at inner radius

m,n representative points

max maximum value due to choking

T station at outer radius

u tangential component

x axial component

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$1, 2, 3 \ldots n$ stations (fig. 1)

0 NACA sea-level air

Superscripts:

$*$ reference line or state

$'$ stagnation state

$''$ stagnation state relative to rotor

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APPENDIX B

CONSTRUCTION OF TYPE-A SPECIFIC DESIGN CHART FOR ZERO EXIT WHIRL

In order to construct a specific design chart for a zero-exit-whirl configuration, the required specific work parameter E must be known. The values of the ratios $a_{cr,3}/a_{cr,1}$ and $\rho_1{}'a_{cr,1}/\rho_3{}'a_{cr,3}$ can be computed from the value of E by use of equations (21) and (22) or found from figures 4 and 5. A range of blade speeds and rotor inlet relative velocities are assigned as critical velocity ratios in the forms $U/a_{cr,1}$ and $W_2/a''_{cr,2}$. For the required value of E, the inlet whirl component can be found from equation (10) with $V_{u,3}/a_{cr,3} = 0$.

$$\frac{V_{u,2}}{a_{cr,1}} = \frac{g\,E}{U/a_{cr,1}} \tag{B1}$$

The rotor inlet relative velocity referred to absolute stagnation temperature can be computed from equations (28) and (B1) as

$$\frac{W_2}{a_{cr,1}} = \frac{W_2}{a''_{cr,2}}\left\{1 - \frac{\gamma-1}{\gamma+1}\left[2g\,E - \left(\frac{U}{a_{cr,2}}\right)^2\right]\right\}^{\frac{1}{2}} \tag{B2}$$

From the trigonometry of the velocity diagram, the absolute velocity at station 2 is

$$\frac{V_2}{a_{cr,1}} = \left[\left(\frac{W_2}{a_{cr,1}}\right)^2 + 2g\,E - \left(\frac{U}{a_{cr,1}}\right)^2\right]^{\frac{1}{2}} \tag{B3}$$

the axial component is

$$\frac{V_{x,2}}{a_{cr,1}} = \left[\left(\frac{V_2}{a_{cr,1}}\right)^2 - \left(\frac{V_u}{a_{cr,1}}\right)^2\right]^{\frac{1}{2}} \tag{B4}$$

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With the known absolute critical velocity ratio, the density ratio ρ_2/ρ_2' can be calculated from

$$\frac{\rho_2}{\rho_2'} = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_2}{a_{cr,1}}\right)^2\right]^{\frac{1}{\gamma-1}}$$

A plot of this function for $\gamma = 1.30$ is given in figure 11. The specific weight-flow parameter $\left(\rho V_x/\rho' a_{cr}\right)_2$ can then be computed.

A flow chart for $\gamma = 1.30$ is included at the back of this report. This chart presents the specific weight-flow parameter as a function of the velocity components.

The specific weight-flow parameter at station 3 can be found for an assigned area ratio by substituting in the following form of equation (17a):

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_3 = \left(\frac{\rho V_x}{\rho' a_{cr}}\right)_2 \left(\frac{\rho_1' a_{cr,1}}{\rho_3' a_{cr,3}}\right) \frac{A_2}{A_3}$$

For this zero-exit-whirl case, $\left(V_u/a_{cr}\right)_3 = 0$ and the exit-velocity ratio can be found by use of the large flow chart.

The exit relative velocity ratio can be computed from

$$\frac{W_3}{a_{cr,3}} = \left[\left(\frac{V_3}{a_{cr,3}}\right)^2 + \left(\frac{U}{a_{cr,1}} \frac{a_{cr,1}}{a_{cr,3}}\right)^2\right]^{\frac{1}{2}}$$

The value of the exit relative velocity ratio based on relative stagnation conditions $W_3/a''_{cr,3}$ can be computed by use of equation (26).

After these components have been computed, the design chart can be drawn. In order to obtain lines of constant velocity ratios not assigned in the computations, it will be necessary to make use of cross plots. The exit critical velocity ratio for zero exit whirl is a function of the specific weight-flow parameter, and an auxiliary scale of $V_3/a_{cr,3}$ can be plotted along side the ordinate.

The envelope, which indicates choking at station 2 (the axial component reaching sonic speed) in the type-A specific design chart, can be found by using equation (B1) to compute the inlet tangential component for a series of assigned blade speeds. The limiting specific weight-flow parameter, for the tangential components thus obtained, can be computed by use of equation (25). 2170

Choking in the annulus at the turbine exit will occur when the exit axial component reaches sonic speed. For the zero-exit-whirl configuration, the exit critical velocity ratio will reach a value of 1.0 at this point.

Equations for the construction of the side plots are given in appendix E.

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APPENDIX C

CONSTRUCTION OF TYPE-B SPECIFIC DESIGN CHART FOR FIXED EXIT CRITICAL VELOCITY RATIO AND VARYING WHIRL

In order to construct the type-B specific design chart, the desired specific work parameter E must be known, and the exit critical velocity ratio $V_3/a_{cr,3}$ must be fixed. The ratios $a_{cr,3}/a_{cr,1}$ and $\rho_1'a_{cr,1}/\rho_3'a_{cr,3}$ can be computed for the given value of E by use of equations (21) and (22) or found from figures 4 and 5.

The ranges of speeds and specific weight-flow parameters are assumed in the following forms:

$$\frac{U}{a_{cr,1}} \quad \text{and} \quad \left(\frac{\rho V_x}{\rho' a_{cr}}\right)_2$$

The flow at station 3 is first determined. The specific weight-flow parameter can be computed for an assigned area ratio by using the following form of equation (17a):

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_3 = \left(\frac{\rho V_x}{\rho' a_{cr}}\right)_2 \frac{\rho_1' a_{cr,1}}{\rho_3' a_{cr,3}} \frac{A_2}{A_3}$$

By use of the flow chart, the computed value of $(\rho V_x/\rho' a_{cr})_3$ and the fixed value of $V_3/a_{cr,3}$, the axial and tangential components can be read.

The inlet tangential component for the required work can be found from the following form of equation (10):

$$\frac{V_{u,2}}{a_{cr,1}} = \frac{g\,E}{U/a_{cr,1}} + \frac{V_{u,3}}{a_{cr,3}} \frac{a_{cr,3}}{a_{cr,1}}$$

In most cases, $V_{u,3}/a_{cr,3}$ should be negative, that is, the exit tangental component is in a direction opposite to the direction of that component at the stator exit.

The absolute and the axial components $V_2/a_{cr,1}$ and $V_{x,2}/a_{cr,1}$ can be obtained by use of a flow chart and the known values of $(\rho V_x/\rho' a_{cr})_2$ and $V_{u,2}/a_{cr,1}$. For each value of $(\rho V_x/\rho' a_{cr})_2$ there are two values of $V_{u,2}/a_{cr,1}$. The higher value is used only when the the axial component of the velocity is greater than sonic speed.

The relative velocity $W_2/a_{cr,1}$ can be computed from equation (27). This value can be corrected to relative stagnation conditions by using either equation (26) or (28). From equation (27)

$$\frac{W_3}{a_{cr,3}} = \left[\left(\frac{V_{x,3}}{a_{cr,3}}\right)^2 + \left(\frac{V_{u,3}}{a_{cr,3}} - \frac{U}{a_{cr,2}} \frac{a_{cr,2}}{a_{cr,3}}\right)^2\right]^{\frac{1}{2}}$$

Angles can be computed from the trigonometry of the velocity diagrams.

These computations supply the needed information for the construction of the specific design chart. In order to plot lines of constant values of parameters such as critical velocity ratios, however, it will be necessary to make use of cross plots, as constant values were not assigned in the computations.

With fixed exit critical velocity ratio, the exit flow angle is a function of the specific weight-flow parameter and an auxiliary scale can be plotted on the ordinate. When desired values of the exit angle α_3 are assigned, the corresponding value of the specific weight-flow parameter at station 3 can be read from the flow chart or computed from

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_3 = \frac{V_3}{a_{cr,3}} \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_3}{a_{cr,3}}\right)^2\right]^{\frac{1}{\gamma-1}} \sin \alpha_3$$

Values of the specific weight-flow parameter at station 2 can be found by use of equation (17a) and a scale for α_3 can be added on the ordinate as shown in figure 8(b).

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The envelope, which indicates choking flow at the rotor inlet, for the type-B specific design charts can be computed as follows:

The exit critical velocity ratio was fixed and the exit-whirl component for the assigned exit angles α_3 can be computed from

$$\frac{V_{u,3}}{a_{cr,3}} = \frac{V_3}{a_{cr,3}} \cos \alpha_3$$

Each assigned value of α_3 has a corresponding value of $(\rho V_x/\rho' a_{cr})_2$, which can readily be found as previously outlined. If each value is considered to be a maximum (that is, $(V_x/a)_2 = 1$) the corresponding tangential component can be found from figure 6. The blade speed for that tangential component can be computed from the following form of equation (10)

$$\frac{U}{a_{cr,1}} = \frac{gE_{1,3}}{\frac{V_{u,2}}{a_{cr,1}} - \frac{V_{u,3}}{a_{cr,3}} \frac{a_{cr,3}}{a_{cr,1}}}$$

The envelope, which indicates limiting flow at the rotor exit, will depend on the assigned value of the rotor exit critical velocity ratio. If the value is less than 1.0, limiting flow occurs when the flow is axial. The value of the specific weight-flow parameter for this case $(\alpha_3 = 90^\circ)$ can be found as previously shown.

If the value of the exit critical velocity ratio is greater than 1.0, choking will occur when the axial velocity reaches sonic speed. By means of the assigned value of $(V/a_{cr})_3$, equations (12) and (23) can be solved simultaneously for the velocity components $(V_u/a_{cr})_3$ and $(V_x/a_{cr})_3$. The specific weight-flow parameter $(\rho V_x/\rho' a_{cr})_3$ can be found from equation (11). The value of $(\rho V_x/\rho' a_{cr})_2$ for the assigned area ratio can be found from equation (17a). This maximum value is the same for all values of blade speed.

APPENDIX D

CONSTRUCTION OF TYPE-C SPECIFIC DESIGN CHART FOR FIXED ROTOR INLET RELATIVE CRITICAL VELOCITY RATIO AND VARYING WHIRL

For the construction of this type of chart, the value of the rotor inlet relative critical velocity ratio $W_2/a''_{cr,2}$ is set and the ranges of blade speeds $U/a''_{cr,2}$ and rotor inlet relative angles β_2 are assigned.

Compute

$$\frac{V_{x,2}}{a''_{cr,2}} = \frac{W_2}{a''_{cr,2}} \sin \beta_2$$

$$\frac{W_{u,2}}{a''_{cr,2}} = \frac{W_2}{a''_{cr,2}} \cos \beta_2$$

$$\frac{V_2}{a''_{cr,2}} = \left[\left(\frac{V_{x,2}}{a''_{cr,2}}\right)^2 + \left(\frac{W_{u,2}}{a''_{cr,2}} + \frac{U}{a''_{cr,2}}\right)^2\right]^{\frac{1}{2}}$$

These velocities, which have been based on relative stagnation conditions, can now be corrected to absolute stagnation conditions by use of the following equation, which was derived from equation (26)

$$\frac{a_{cr,2}}{a''_{cr,2}} = \left\{1 - \frac{\gamma-1}{\gamma+1}\left[\left(\frac{W_2}{a''_{cr,2}}\right)^2 - \left(\frac{V_2}{a''_{cr,2}}\right)^2\right]\right\}^{\frac{1}{2}} \tag{D1}$$

The values of $V_2/a_{cr,2}$, $V_{x,2}/a_{cr,2}$, and $U/a_{cr,2}$ can be computed and the specific weight-flow parameter $(\rho V_x/\rho' a_{cr})_2$ can then be found.

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From equation (17a), for an assigned area ratio,

$$\left(\frac{\rho V_x}{\rho' a_{cr}}\right)_3 = \left(\frac{\rho V_x}{\rho' a_{cr}}\right)_2 \frac{\rho'_1 a_{cr,1}}{\rho'_3 a_{cr,3}} \frac{A_2}{A_3} \tag{D2}$$

The tangential component can be found from a form of equation (10)

$$\frac{V_{u,3}}{a_{cr,3}} = \frac{a_{cr,1}}{a_{cr,3}}\left(\frac{V_{u,2}}{a_{cr,1}} - \frac{g\,E_{1,3}}{U/a_{cr,1}}\right)$$

For the known values of $(\rho V_x/\rho' a_{cr})_3$ and $(V_u/a_{cr})_3$, the values of the other velocity components may be read from a flow chart (fig. 3) and the design chart can be plotted. In order to plot lines of constant values of the parameters, it is necessary to use cross plots, because constant values of these parameters were not assigned in the computations.

The envelope at the inlet to the rotor in type-C specific design charts will depend on the assigned value of the rotor inlet relative critical velocity ratio. If the value of this fixed critical velocity ratio is less than 1.0, limiting flow will occur when the specific weight flow $(\rho V_x/\rho' a_{cr})_2$ reaches its limiting value. This value can be found by expressing $(\rho V_x/\rho' a_{cr})_2$ as a function of blade speed $U/a_{cr,1}$ and the tangential component of the inlet relative critical velocity ratio $(W_u/a_{cr})_2$. Differentiating this expression with respect to $(W_u/a_{cr})_2$ and setting the derivative equal to zero results in an implicit relation between $U/a_{cr,1}$ and $(W_u/a_{cr})_2$. Theoretically, it will be then possible to express the limiting value of $(\rho V_x/\rho' a_{cr})_2$ as a function of blade speed $U/a_{cr,1}$ alone. Practically, such an expression for the envelope is difficult to obtain. The line of $\beta_2 = 90^\circ$ can be used as a very close approximation to this envelope at the inlet.

In the case where the value of the fixed rotor inlet relative critical velocity ratio is greater than 1.0, the envelope, which indicates choking at the inlet (station 2), can be found as follows:

By assigning values of the specific weight-flow parameter at station 2 and considering these to be maximum values (that is, the value at which the axial component is equal to sonic speed), the corresponding tangential component of the critical velocity ratio can be found by use of equation (25) or read from figure 6. The axial component for this condition can be found from equation (23) 2170

$$\left(\frac{V_x}{a_{cr}}\right) = \left[1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_u}{a_{cr}}\right)^2\right]^{\frac{1}{2}}$$

The absolute velocity $V_2/a_{cr,2}$ is the square root of the sum of the squares of the components. The value of the fixed rotor inlet relative critical velocity ratio was based on relative stagnation conditions. The value can be referred to absolute stagnation conditions by use of the following equations:

$$\frac{a_{cr,2}}{a''_{cr,2}} = \left[\frac{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{W_2}{a''_{cr,2}}\right)^2}{1 - \frac{\gamma-1}{\gamma+1}\left(\frac{V_2}{a_{cr,2}}\right)^2}\right]^{\frac{1}{2}}$$

and

$$\frac{W_2}{a_{cr,2}} = \frac{W_2}{a''_{cr,2}} \frac{a''_{cr,2}}{a_{cr,2}}$$

The relative tangential component can be obtained from

$$\frac{W_{u,2}}{a_{cr,2}} = \left[\left(\frac{W_2}{a_{cr,2}}\right)^2 - \left(\frac{V_{x,2}}{a_{cr,2}}\right)^2\right]^{\frac{1}{2}}$$

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and the blade speed corresponding to the choking condition will be

$$\frac{U}{a_{cr,2}} = \frac{V_{u,2}}{a_{cr,2}} - \frac{W_{u,2}}{a_{cr,2}}$$

The envelope that occurs as a result of choking in the rotor exit occurs when the axial velocity at station 3 reaches sonic speed. This envelope must be determined by a trial-and-error process.

A value of the exit tangential component $V_{u,3}/a_{cr,3}$ is assigned and the corresponding $(\rho V_x/\rho' a_{cr})_{3,max}$ value is obtained from equation (25) or read from figure 6. For the assigned value of the area ratio and the work parameter, $(\rho V_x/\rho' a_{cr})_2$ can be computed from equation (D2).

A trial-and-error process must then be used to determine the blade speed that corresponds to the assigned $V_{u,3}/a_{cr,3}$.

The fixed inlet relative critical velocity ratio can be related to the absolute velocity components by using equations (27) and (28).

$$\left(\frac{W_2}{a''_{cr,2}}\right)^2 \left\{1 - \frac{\gamma-1}{\gamma+1}\left[2\,\frac{V_{u,2}}{a_{cr,2}}\,\frac{U}{a_{cr,1}} - \left(\frac{U}{a_{cr,1}}\right)^2\right]\right\}$$

$$= \left(\frac{V_{x,2}}{a_{cr,2}}\right)^2 + \left(\frac{V_{u,2}}{a_{cr,2}} - \frac{U}{a_{cr,1}}\right)^2 \qquad \text{(D3)}$$

With various values of blade speed $U/a_{cr,1}$ and the assigned value of $V_{u,3}/a_{cr,3}$ equation (10) can be solved for $V_{u,2}/a_{cr,1}$

By using the values of $U/a_{cr,1}$ and the corresponding values of $V_{u,2}/a_{cr,1}$ equation (D3) can be solved for $V_{x,2}/a_{cr,2}$.

When all the velocity components at station 2 are known, equation (11) can be solved for $(\rho V_x/\rho' a_{cr})_2$. The values of $(\rho V_x/\rho' a_{cr})_2$ thus obtained can be plotted against the assigned values $U/a_{cr,1}$. By means of this curve, the blade speed that corresponds to the maximum specific weight-flow parameter for the assigned $V_{u,3}/a_{cr,3}$ can be 2170
found. If this maximum value of specific weight-flow parameter does not fall on the curve, choking has occurred at the rotor inlet, and no point can be obtained on the envelope by this method for the assigned value of $V_{u,3}/a_{cr,3}$.

This procedure must be repeated for each point on the envelope, and a different value of $V_{u,3}/a_{cr,3}$ must be assigned for each point.

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APPENDIX E

CONSTRUCTION OF SIDE PLOTS

Mass-flow scale. - For a given mass flow per unit frontal area based on tip diameter

$$\frac{w}{\pi r_T^2} = \frac{\rho V_x A}{\pi r_T^2} \tag{E1}$$

Also

$$\frac{\pi r_T^2}{A} = \frac{1}{1 - \left(\frac{r_h}{r_T}\right)^2} \tag{E2}$$

The following equation was obtained by solving equation (E1) for the mass-flow parameter, substituting equation (E2) into equation (E1), and dividing through by $\rho' a_{cr}$:

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{w}{\pi r_T^2} \frac{1}{1 - \left(\frac{r_h}{r_T}\right)^2} \frac{1}{\rho' a_{cr}} \tag{E3}$$

A weight flow corrected to compressor-inlet conditions can be used if desired. In such a case

$$\frac{\rho V_x}{\rho' a_{cr}} = \frac{W \frac{\sqrt{\theta_c}}{\delta_c}}{\pi r_T^2} \frac{1}{1 - \left(\frac{r_h}{r_T}\right)^2} \frac{\delta_c/\sqrt{\theta_c}}{\rho' a_{cr}}$$

With the known turbine inlet total temperature and pressure, $\rho' a_{cr}$ can be computed; for a range of weight flows per unit frontal area and radius ratios the values of the specific weight-flow parameter can be computed, and the side plots that appear at the left of the specific design charts can be plotted.

<u>Blade-tip speed and stress plot</u>. - For a given range of blade-tip speeds and radius ratios, the value of the blade-speed parameter at the hub can be obtained from

$$\frac{U_h}{a_{cr}} = U_T \frac{r_h}{r_T} \frac{1}{a_{cr}}$$

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By using the assigned values of stress, blade density and taper factor, and radius ratio, equation (4) can be solved for the tip speed

$$U_T = \left\{ \frac{\sigma}{\rho_b \tau} \frac{2g\ 144}{\left[1 - \left(\frac{r_h}{r_T}\right)^2\right]} \right\}^{\frac{1}{2}}$$

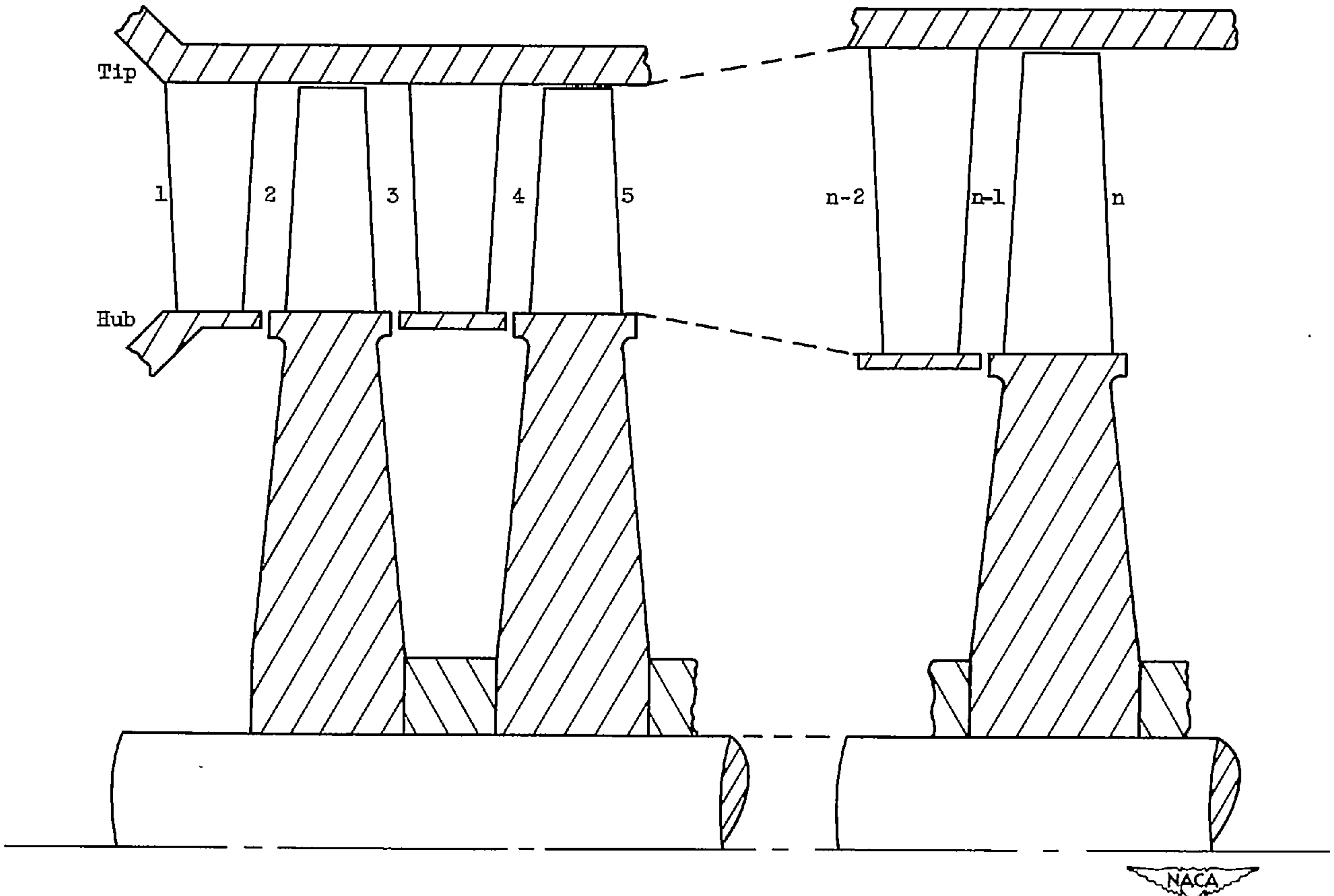
With these values the side plots that appear under the abscissa of the specific design charts can be plotted.

For multistage turbines, where each stage has the same value of E and is of the same type, the same main plot can be used and a separate set of side plots must be plotted for each stage.

REFERENCES

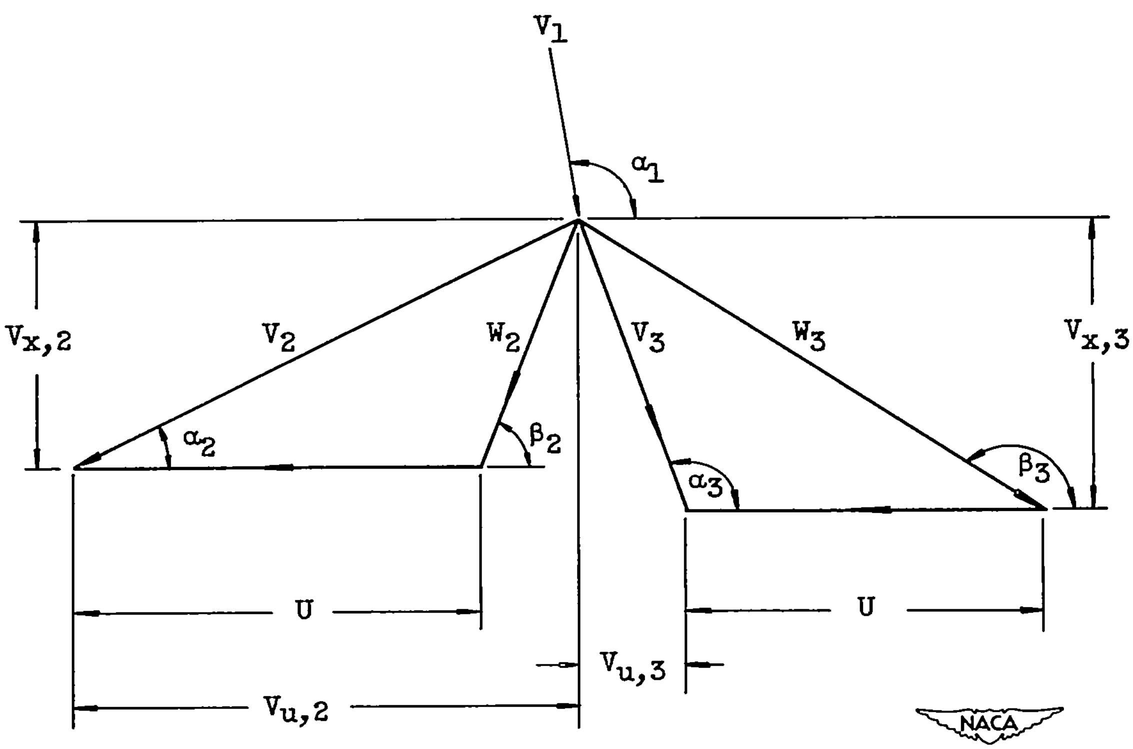
1. The Staff of the Ames 1- by 3-Foot Supersonic Wind-Tunnel Section: Notes and Tables for Use in the Analysis of Supersonic Flow. NACA TN 1428, 1947.

2. Lavalle, Vincent L., and Huppert, Merle C.: Effects of Several Design Variables on Turbine-Wheel Weight. NACA TN 1814, 1949.



(a) Cross-sectional sketch of typical turbine showing flow stations.

Figure 1. - Illustrations showing notation used.



(b) Velocity diagram for stator and rotor components of stage.

Figure 1. - Concluded. Illustrations showing notation used.

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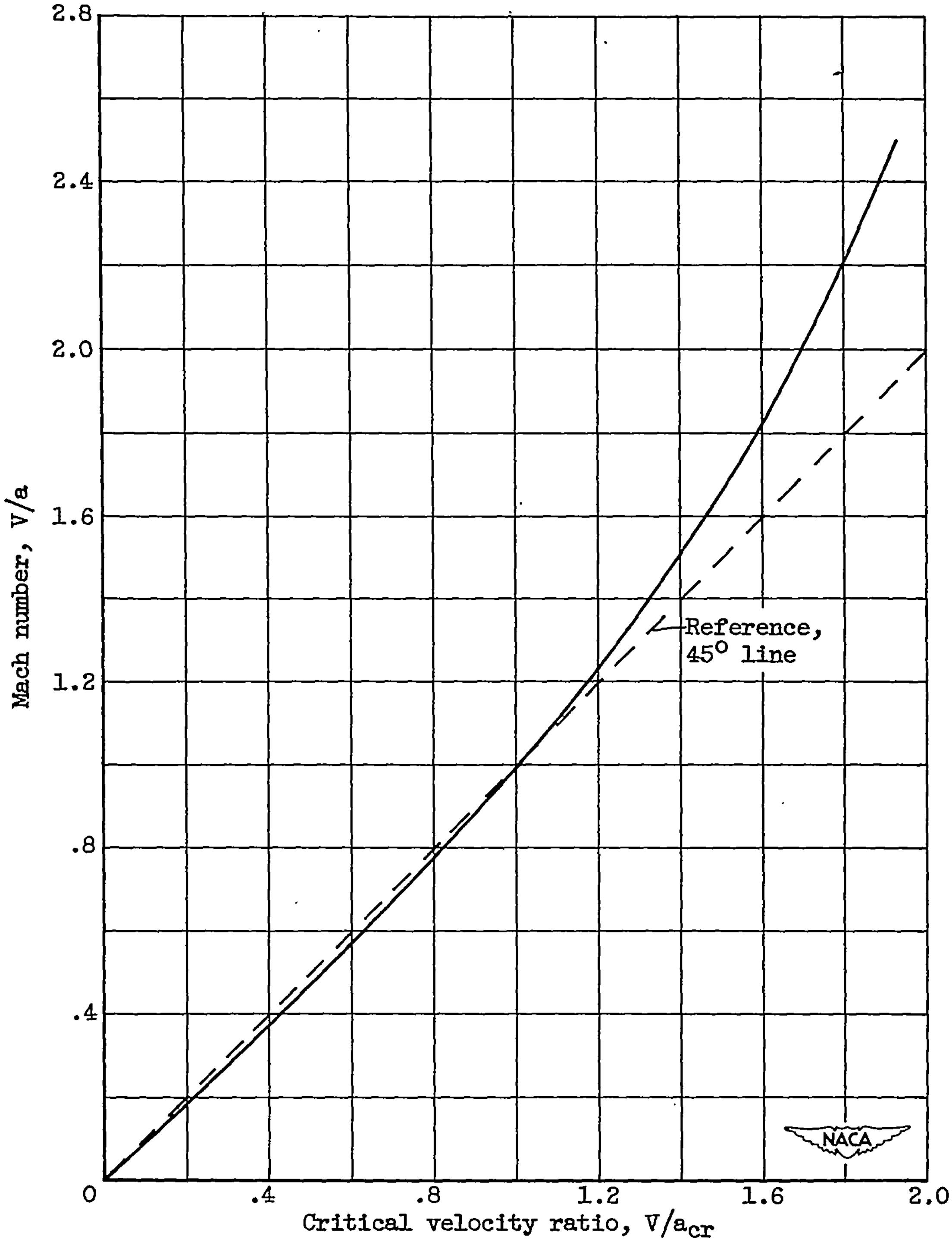

Figure 2. - Variation of Mach number with critical velocity ratio. Ratio of specific heats, 1.30.

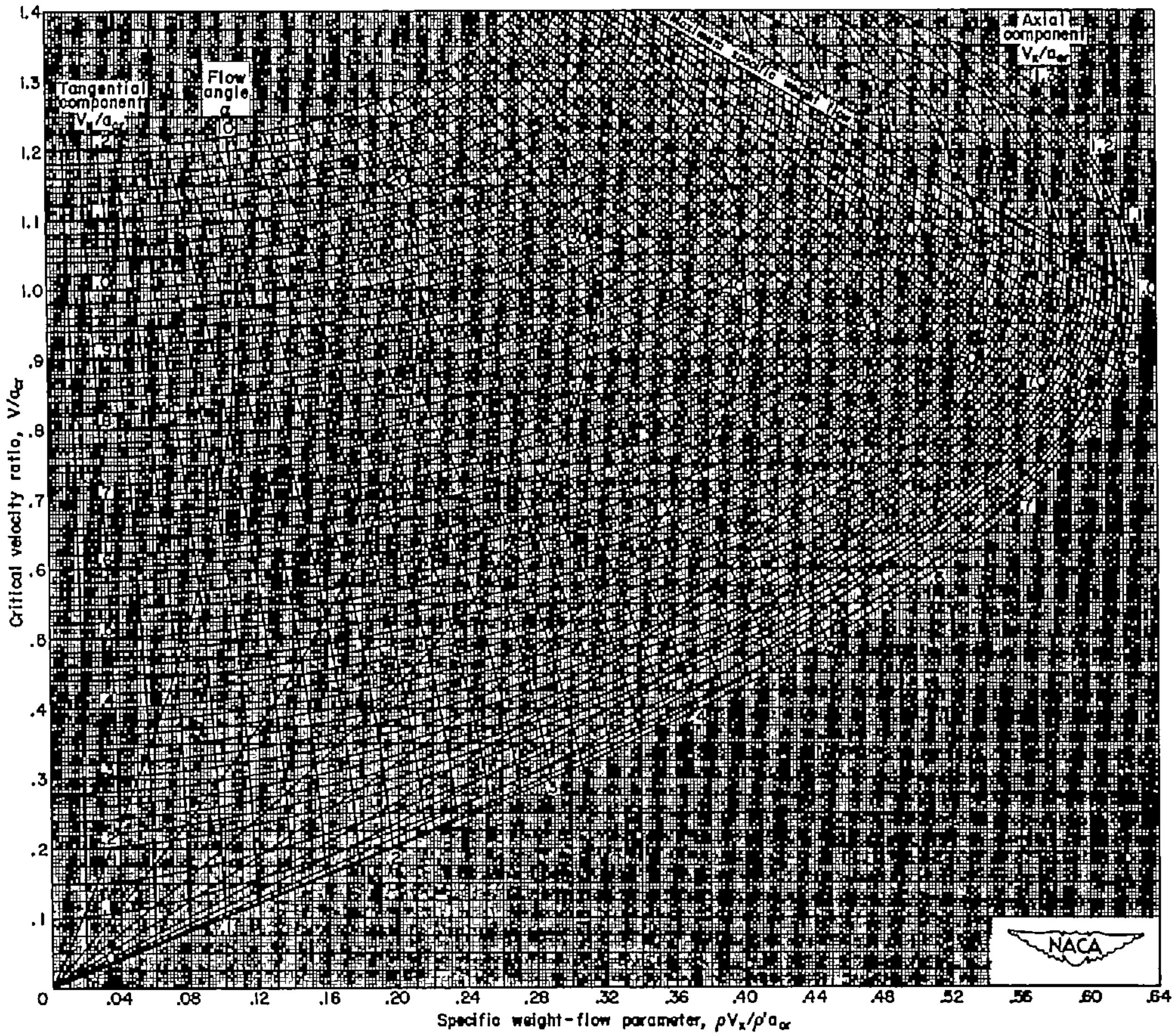

Figure 3. - Flow chart; ratio of specific heats, 1.30. (A 16- by 18-in. print of this fig. is attached.)

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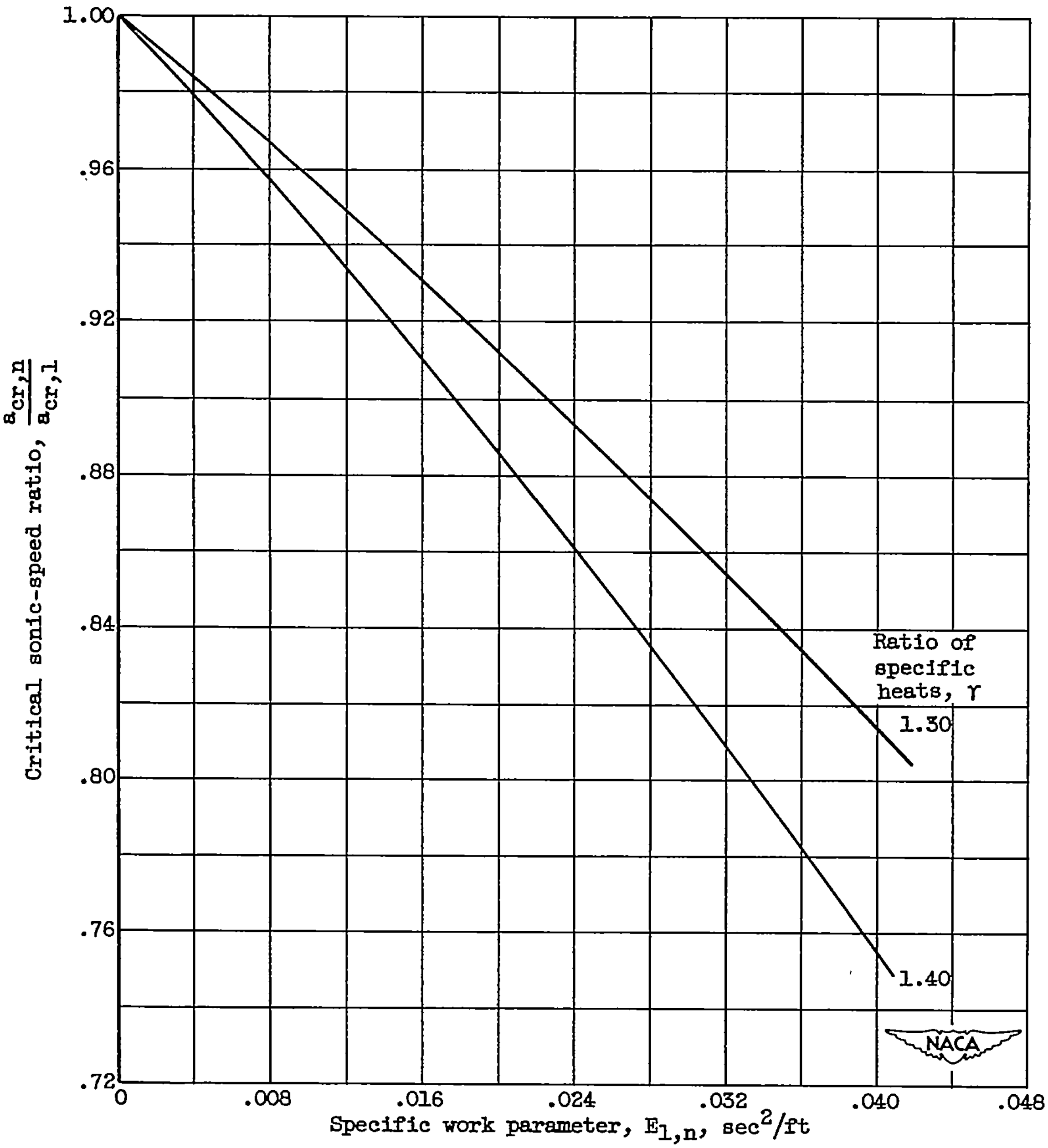


Figure 4. - Variation of critical sonic-speed ratio with specific work parameter for two ratios of specific heats.

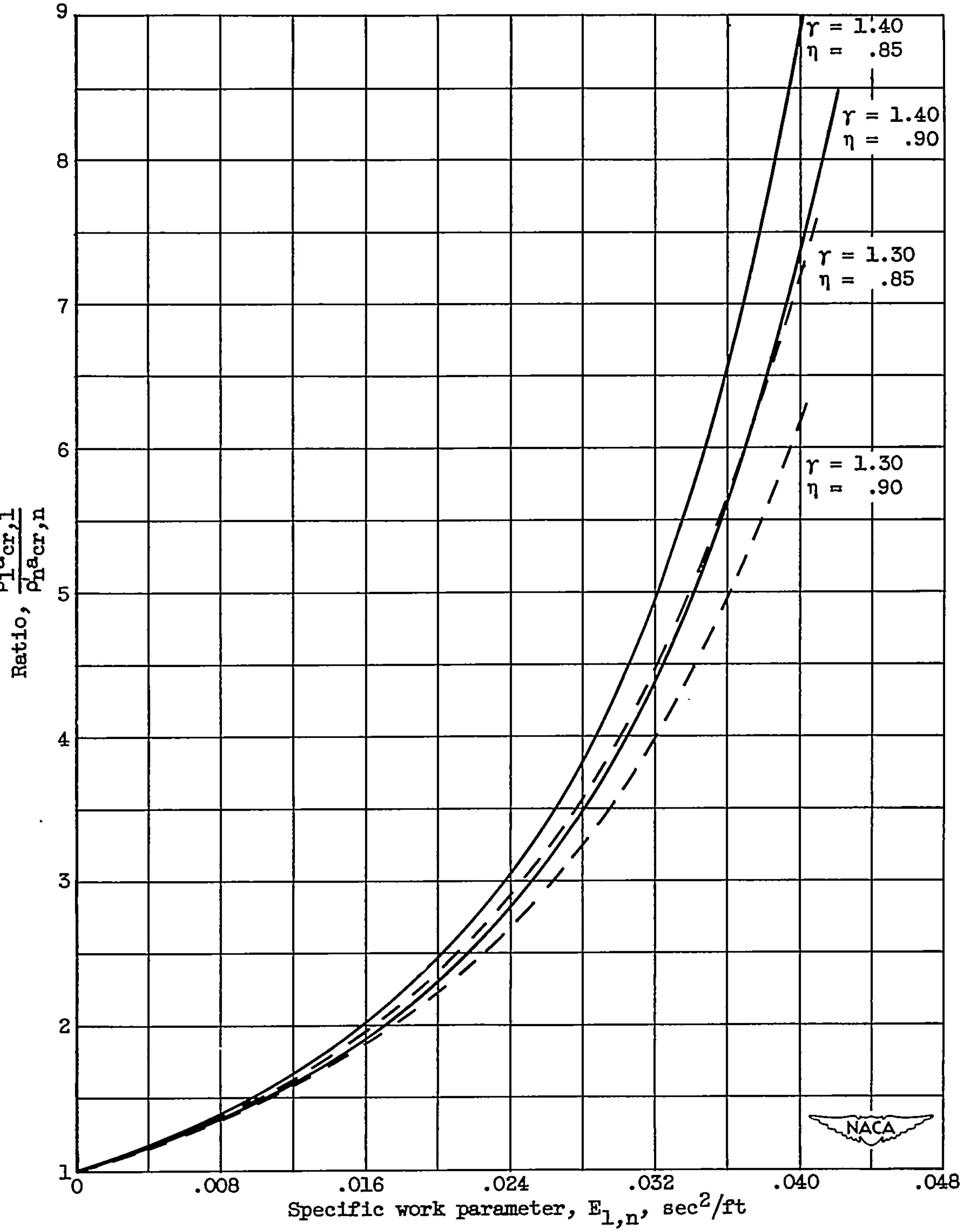


Figure 5. - Variation of $\rho'_1 a_{cr,1}/\rho'_n a_{cr,n}$ with specific work parameter for various values of adiabatic efficiency and ratio of specific heats.

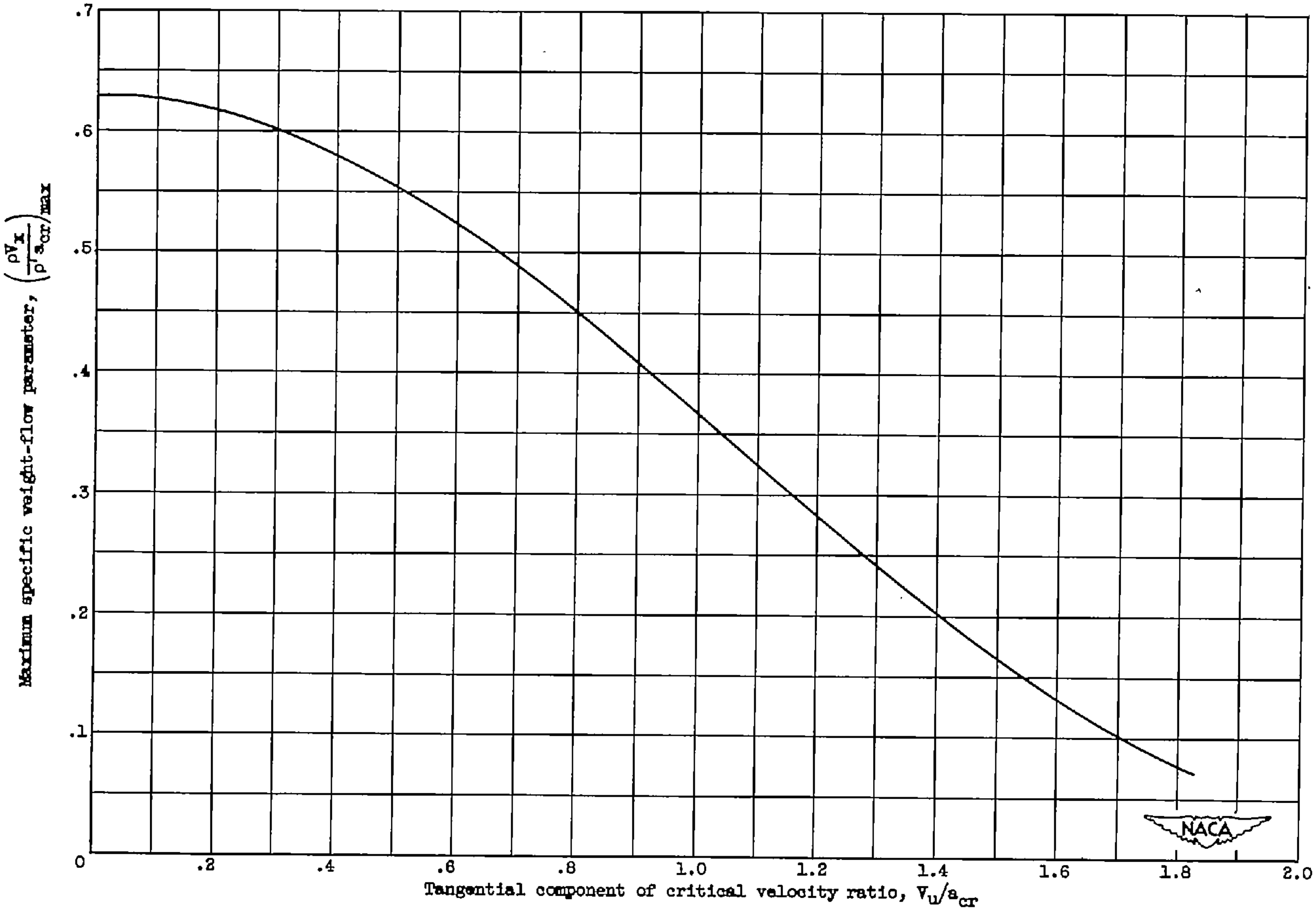
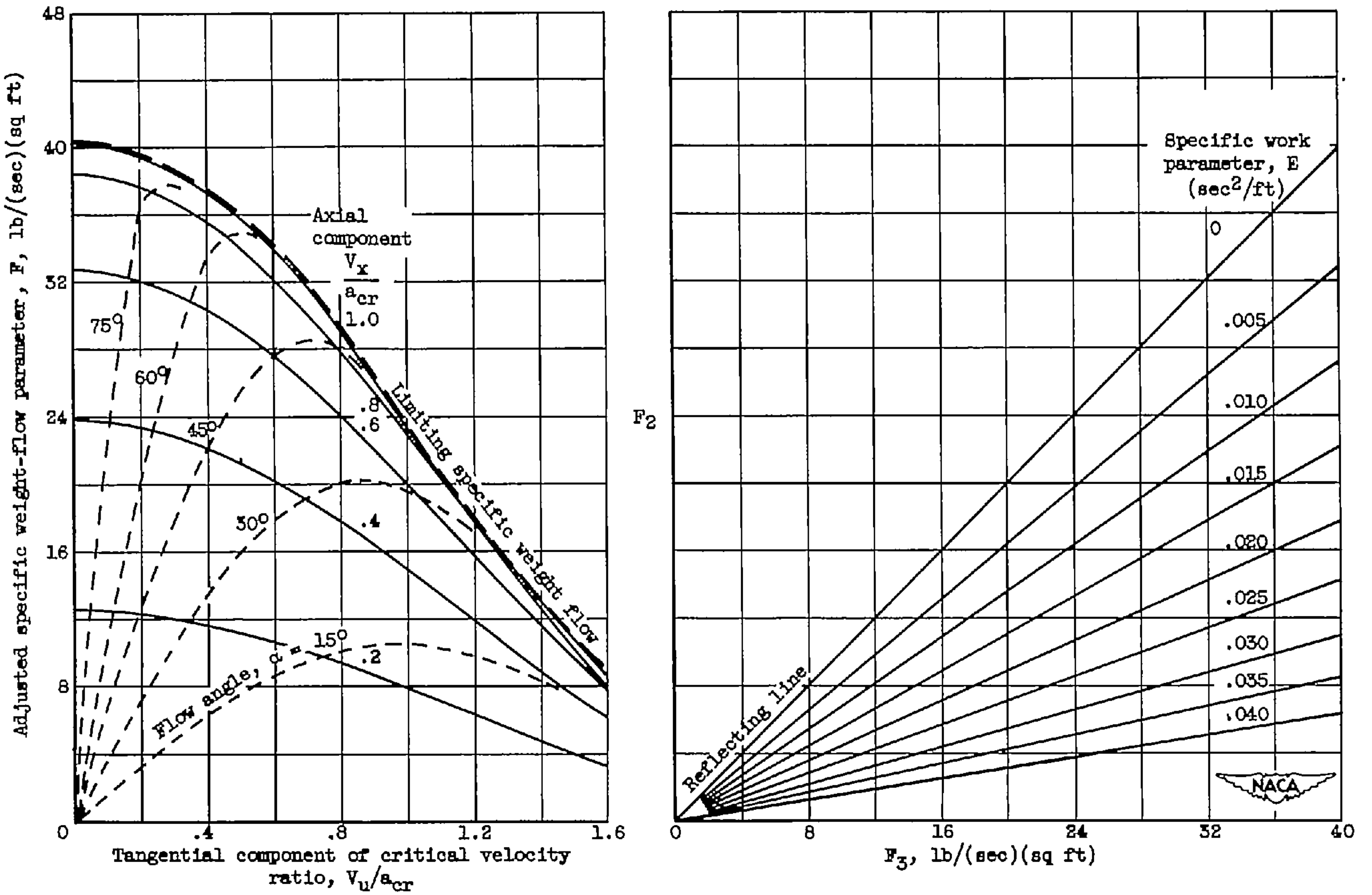
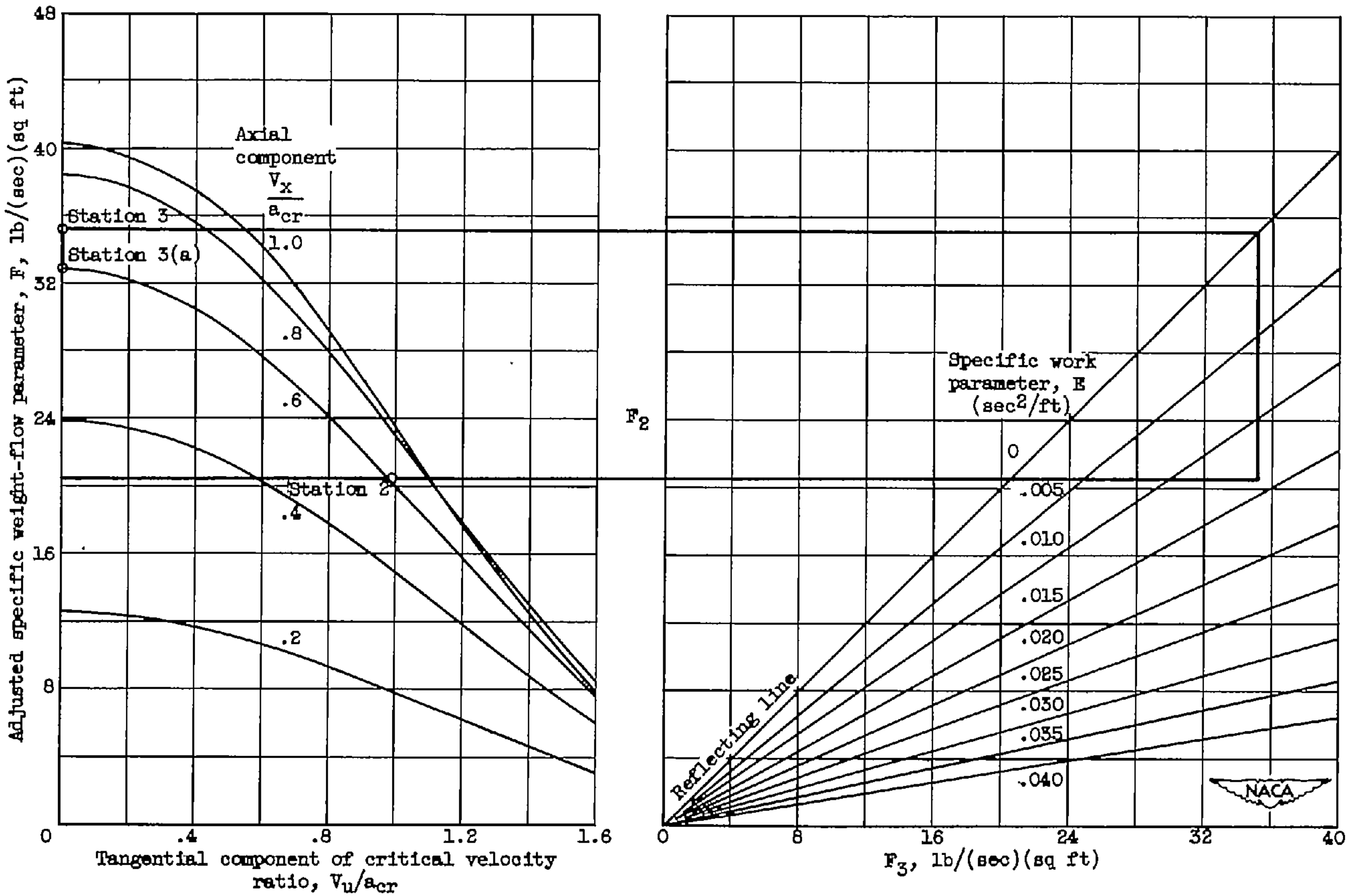


Figure 6. - Plot of maximum specific weight-flow parameter against tangential component of critical velocity ratio. Ratio of specific heats, 1.30.



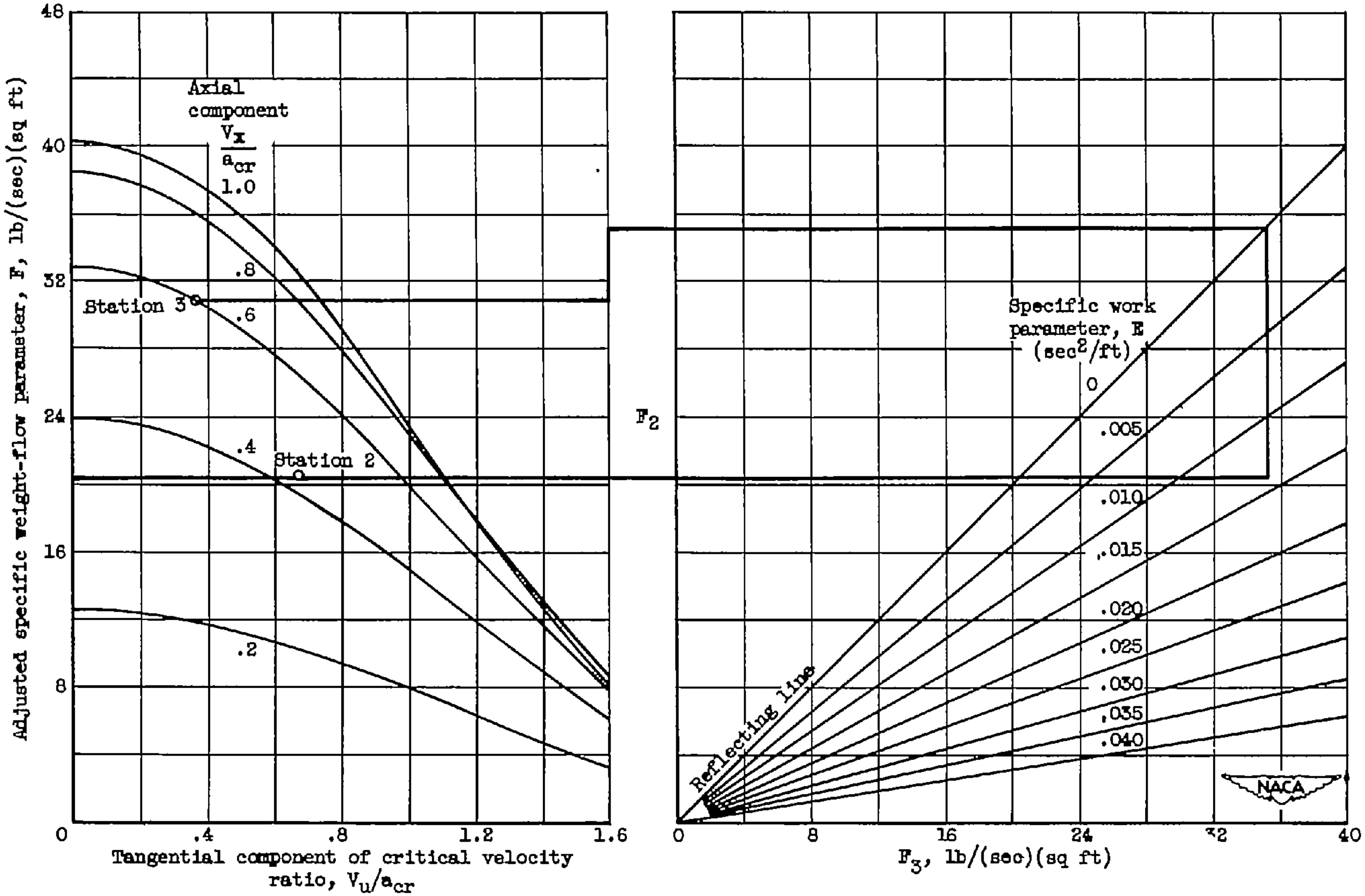
(a) Typical preliminary design chart.

Figure 7. - Preliminary design charts. Ratio of specific heats, 1.30; adiabatic efficiency, 0.90.



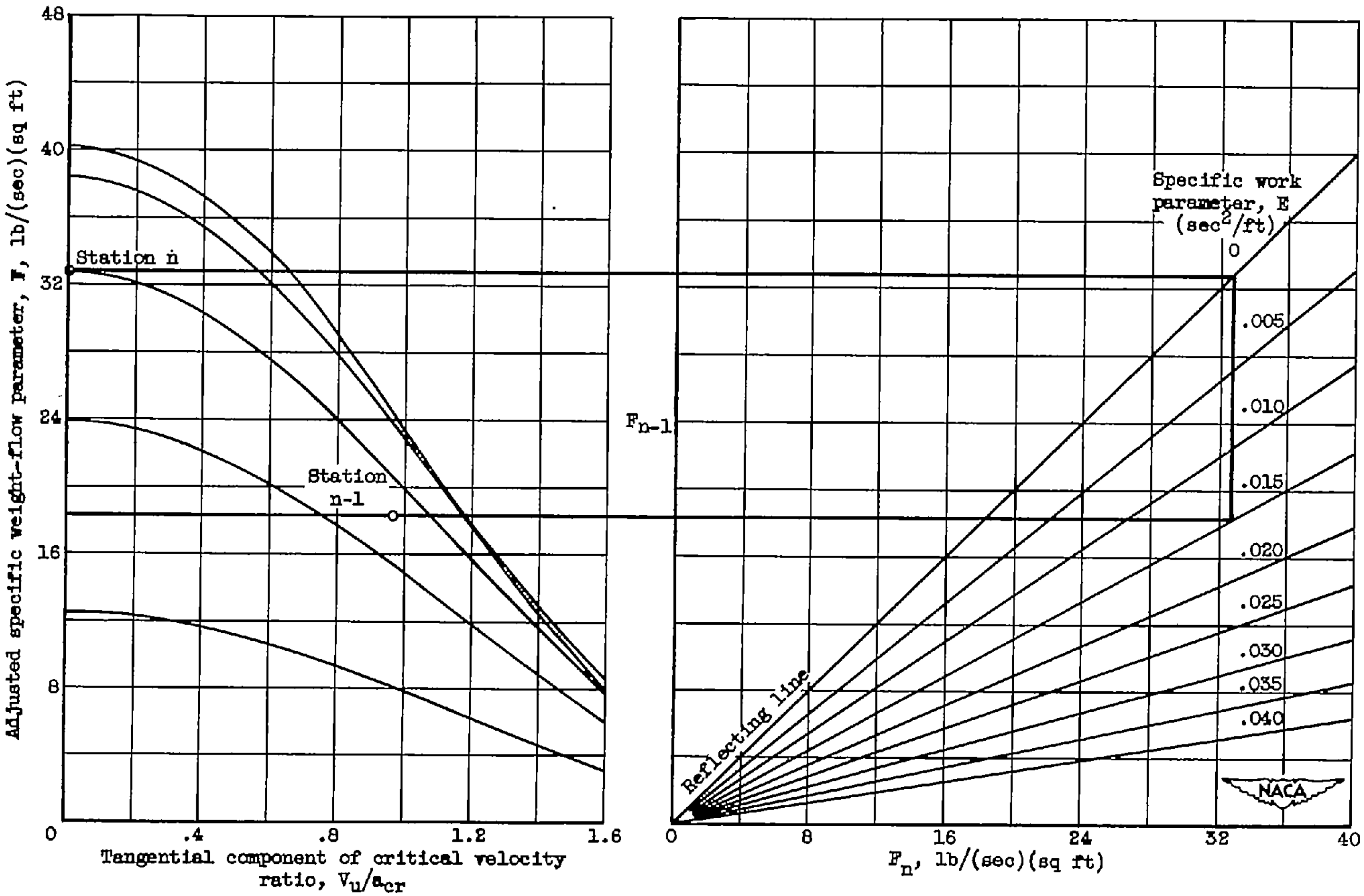
(b) Illustrative example I - analysis of zero-exit-whirl configuration.

Figure 7. - Continued. Preliminary design charts. Ratio of specific heats, 1.30; adiabatic efficiency, 0.90.



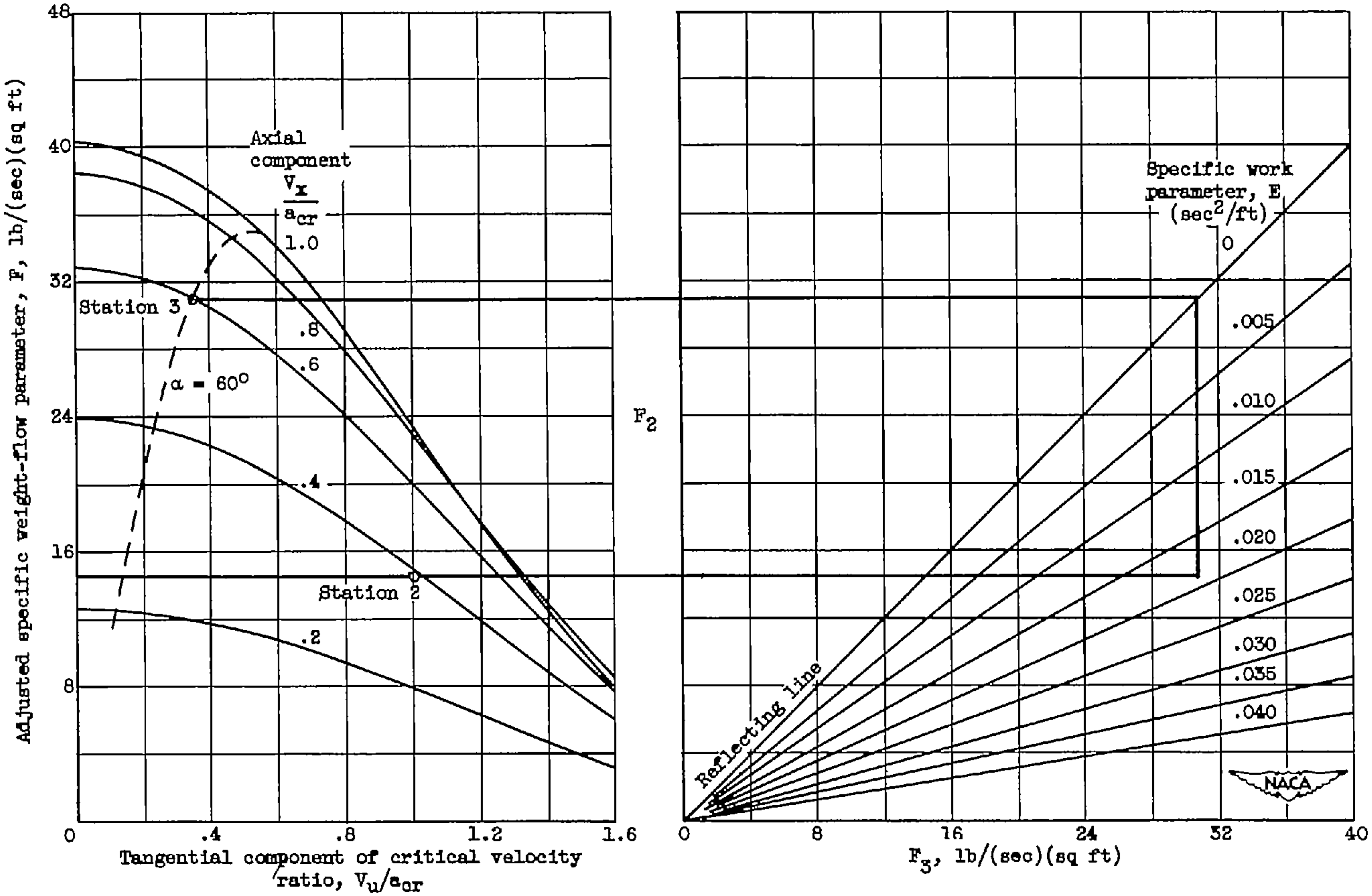
(c) Illustrative example II - analysis of exit-whirl configuration.

Figure 7. - Continued. Preliminary design charts. Ratio of specific heats, 1.30; adiabatic efficiency, 0.90.



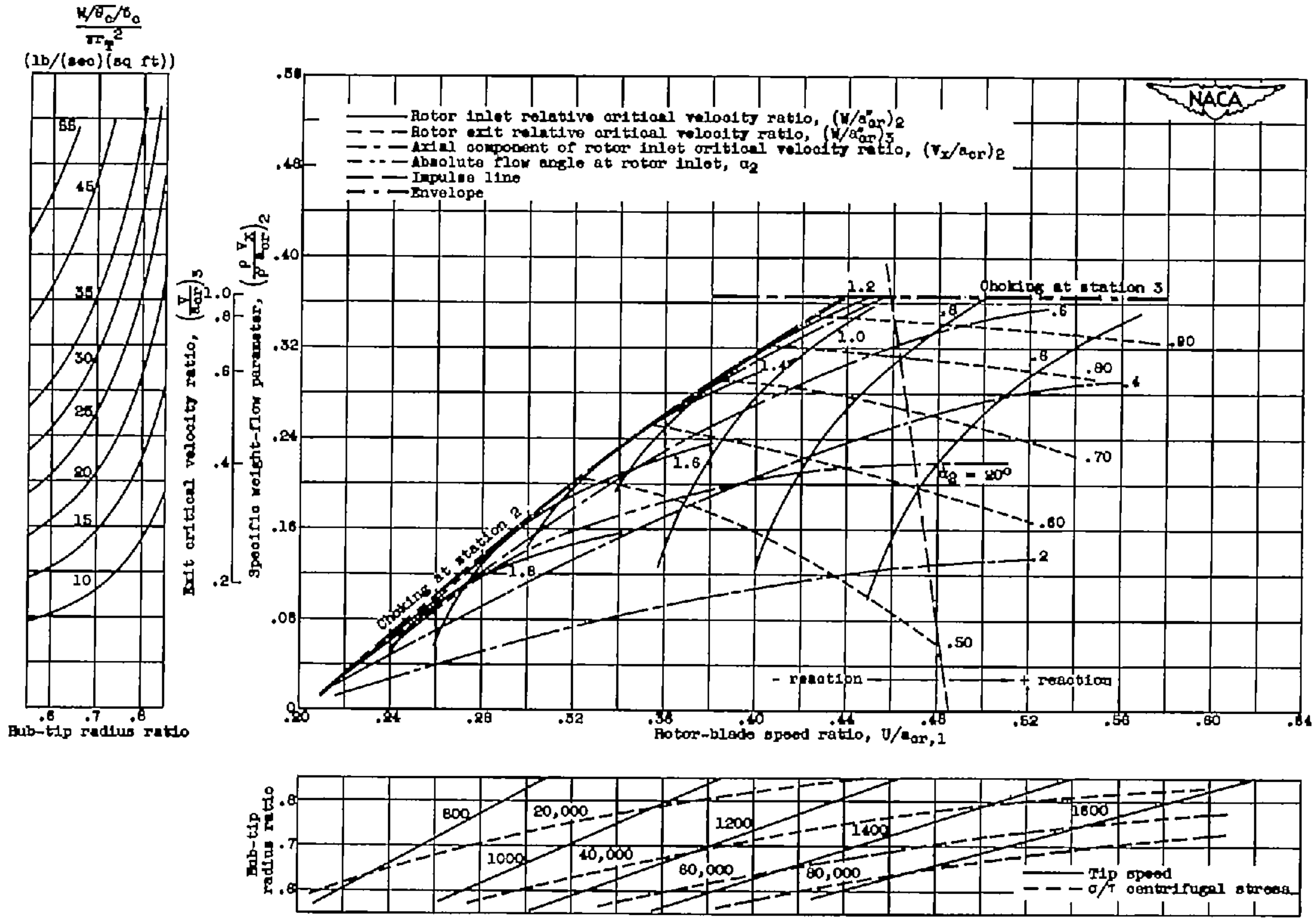
(d) Illustrative example III - analysis of last stage of multistage turbine.

Figure 7. - Continued. Preliminary design charts. Ratio of specific heats, 1.30; adiabatic efficiency, 0.90.



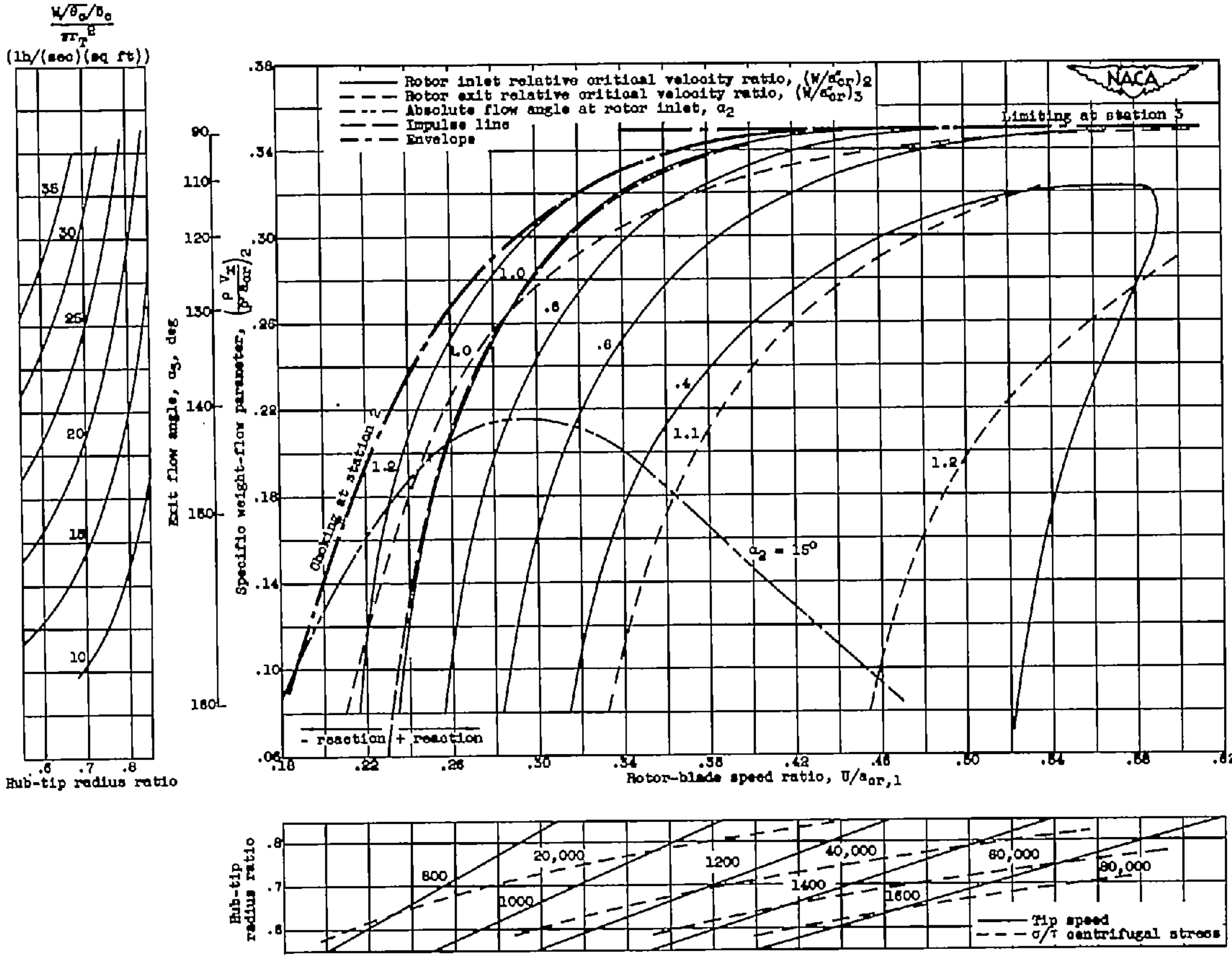
(e) Illustrative example III - analysis of first stage of multistage turbine.

Figure 7. - Concluded. Preliminary design charts. Ratio of specific heats, 1.30; adiabatic efficiency, 0.90.



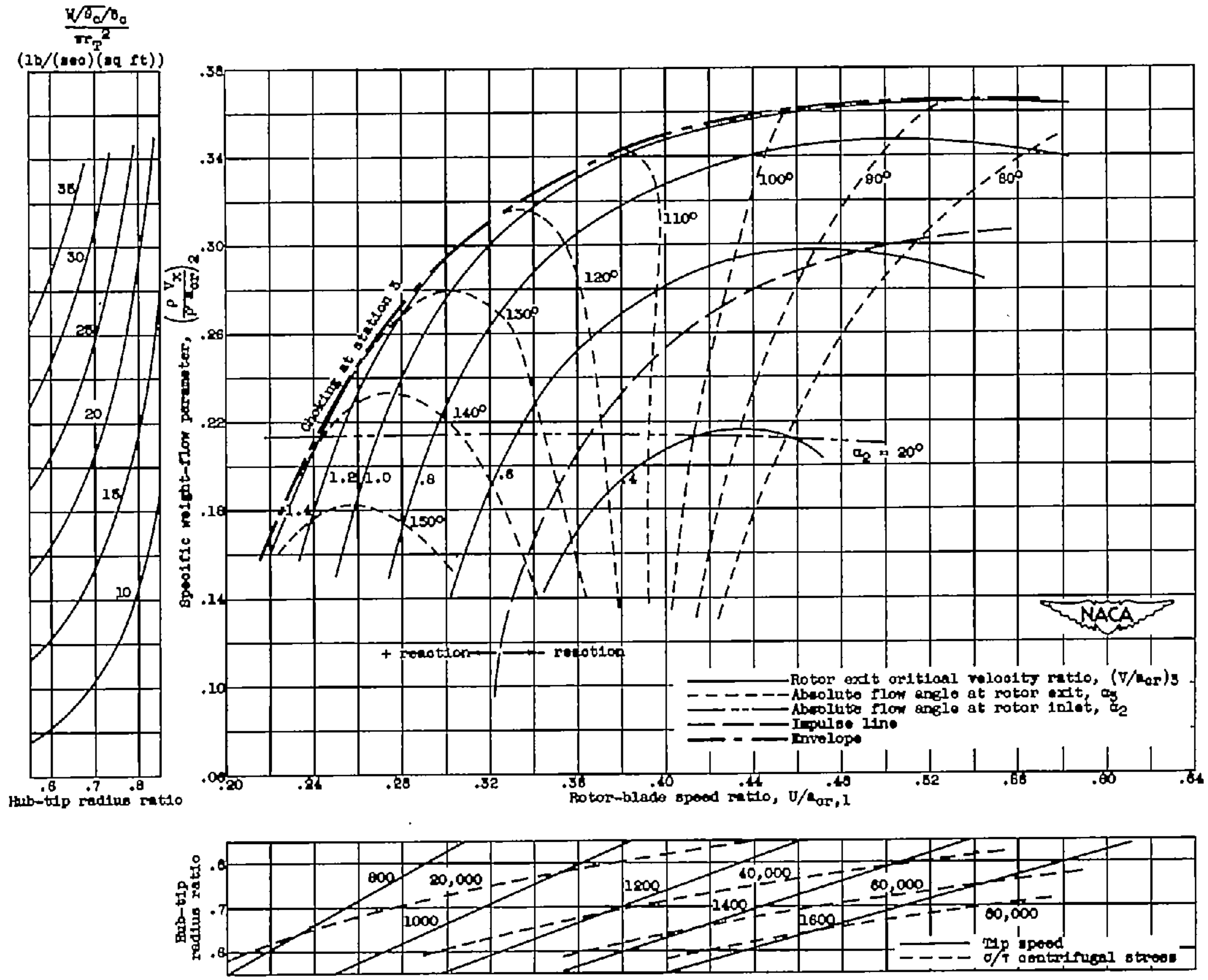
(a) Type A - Zero exit whirl.

Figure 8. - Specific design charts. Specific work parameter, 0.140; turbine-inlet total temperature, 2500° R; turbine-inlet total pressure, 5120 pounds per square foot absolute; ratio of specific heats, 1.30; adiabatic efficiency, 0.90; annular area, constant.



(b) Type B - Exit critical velocity ratio of 0.8 and variable exit whirl.

Figure 8. - Continued. Specific design charts. Specific work parameter, 0.140; turbine-inlet total temperature, 2500° R; turbine-inlet total pressure, 5120 pounds per square foot absolute; ratio of specific heats, 1.30; adiabatic efficiency, 0.90; annular area, constant.



(c) Type C - Inlet critical velocity ratio of 0.75 and variable exit whirl.

Figure 8. - Concluded. Specific design charts. Specific work parameter, 0.140; turbine-inlet total temperature, 2500° R; turbine-inlet total pressure, 5120 pounds per square foot absolute; ratio of specific heats, 1.30; adiabatic efficiency, 0.90; annular area, constant.

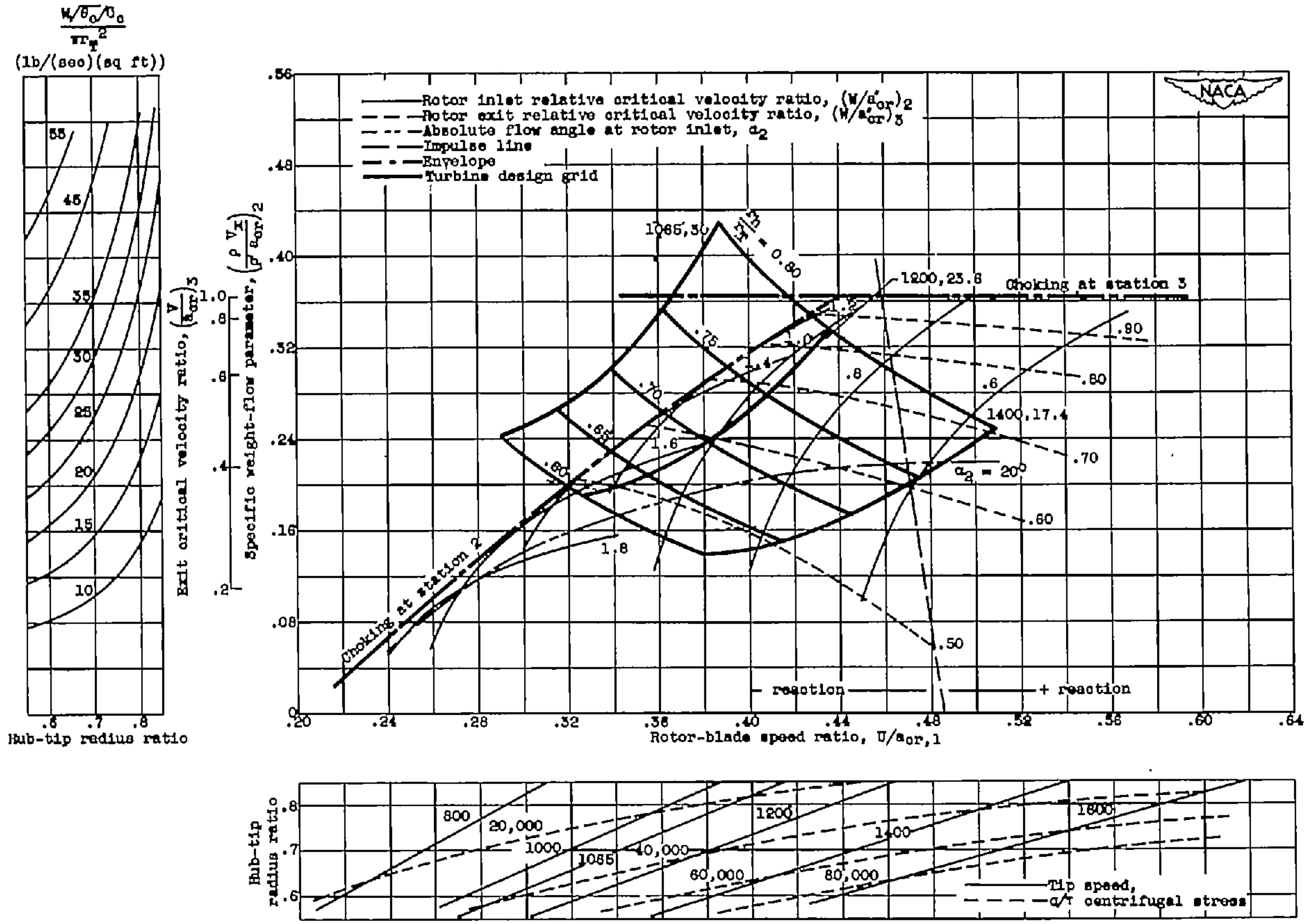


Figure 9. - Illustrative example IV - analysis of single-stage turbine with zero-exit whirl by use of type-A specific design chart. Specific work parameter, 0.140; turbine inlet total temperature, 2500° R; turbine inlet total pressure, 3120 pounds per square foot absolute; ratio of specific heats, 1.30; adiabatic efficiency, 0.90; annular area, constant.

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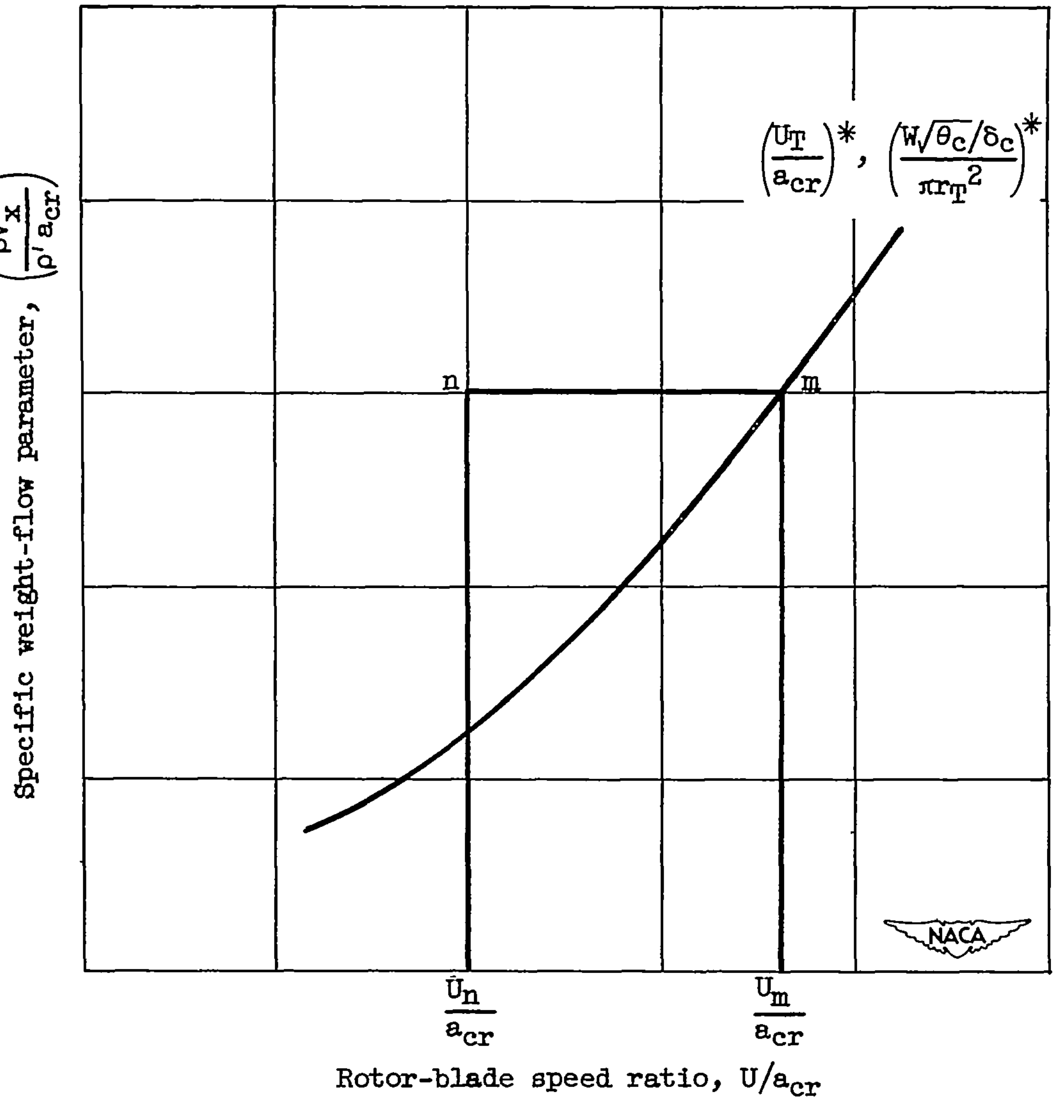

Figure 10. - Section of specific design chart illustrating nomenclature of equations (32), (33), and (34).

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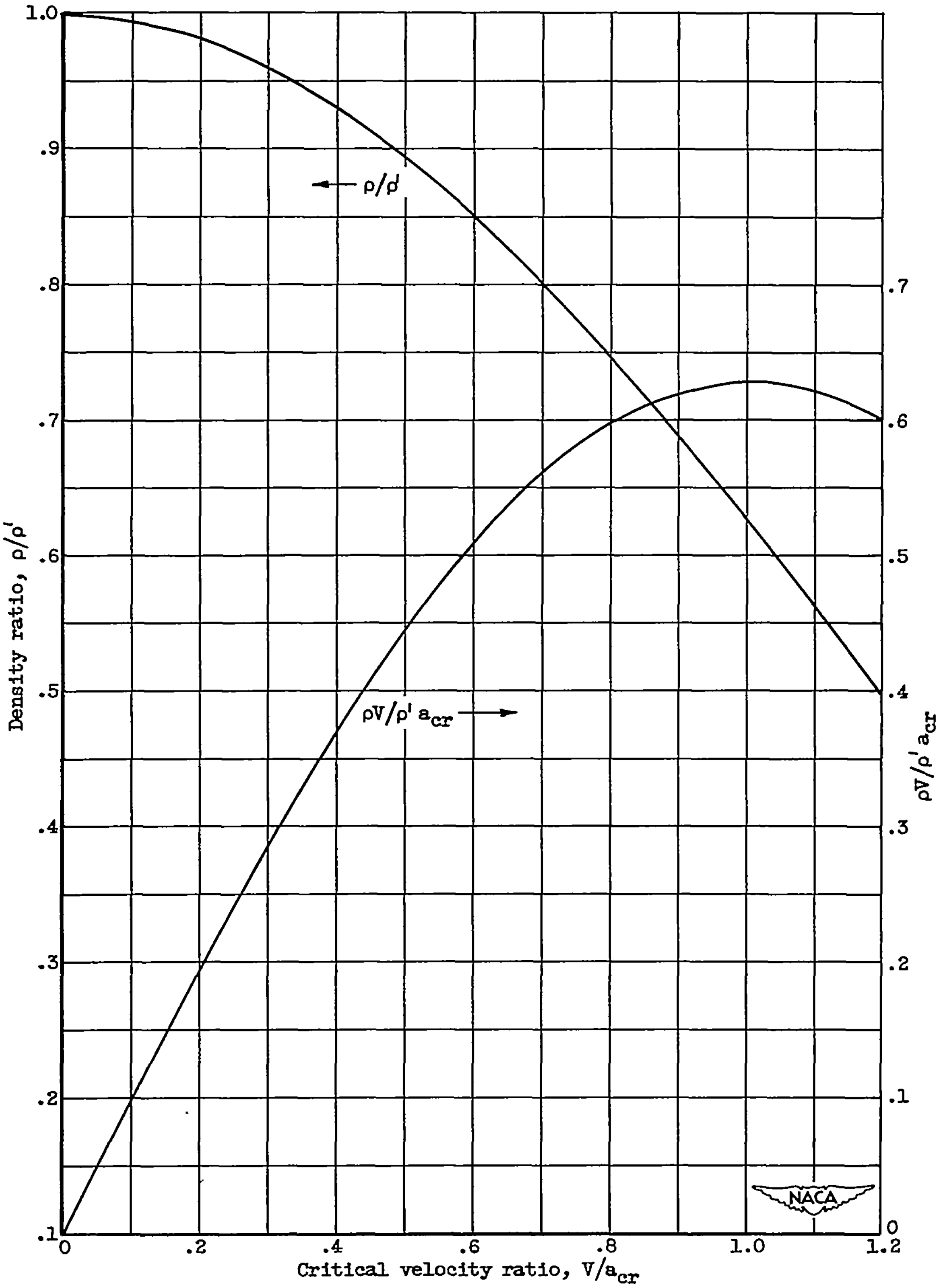

Figure 11. - Plot of density ratio and product of density ratio and critical velocity ratio as function of critical velocity ratio. Ratio of specific heats, 1.30.